# KAPITEL 4 / *CHAPTER 4* [26]
# TECHNIQUE FOR CALCULATING THE PARAMETERS OF THERMOELECTRIC HEAT FLUX SENSORS

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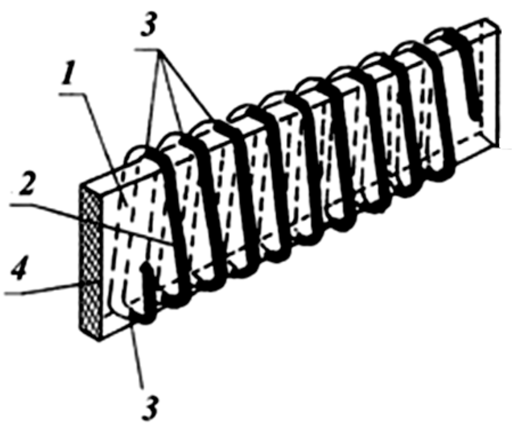
**Introduction.**

To measure the heat flux and its density, the most developed and widely used in practice are multielement thermoelectric heat flux sensors (HFS) of the form of an additional wall, which are a kind of gradient converters [1–3]. Such a converter is a plate in which a battery of thermoelements is placed, filled with an electrical insulating compound.

The battery of thermoelements is made in the form of a tape-like spiral from the wire of the main thermoelectrode, which is wound on a frame electrical insulating tape (figure 1). Moreover, one side of the main wire helix is coated with another (paired) thermoelectric material. The borders of the transition from the section with a pure wire to the bimetallic section are thermoelement junctions, and one of the junctions of each thermoelement is located near one of the HFS surfaces, and the second junction is near the opposite one.




*1 - main thermoelectrode; 2 - bimetal thermoelectrode; 3 - junction of the sections; 4 - electrical insulating tape.*

**Fig.1. Batteries of thermoelements with galvanic coating of thermoelectric material**

Thus, in the presence of a heat flow passing through the HFS plate and, accordingly, in parallel through all elements of the thermopile, a temperature difference occurs between the junctions $\Delta T$, as a result of which, in each of the thermoelements

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connected in series with a thermoelectric coefficient $\alpha_{te}$, an electrical signal $\Delta U$ is generated. If the length of the section with a pure wire and the bimetallic section are equal, the relation is valid [1, 3]:

$$\Delta U = \alpha_{\text{te}} \cdot \Delta T = (\alpha_1 - \alpha_2) \cdot \Delta T \Big/ \left(1 + \frac{\rho_2}{\rho_1} \cdot \frac{f_1}{f_2}\right), \tag{1}$$

where $\alpha_i$ ($i$=1,2) – Seebeck coefficients of main thermoelectrode – wire ($i$=1) and electroplated material ($i$=2); $f_i$ – cross-sectional area of the material; $\rho_i$ – specific electrical resistance of the material.

The main characteristic of HFS is the sensitivity $S$ to the measured value: heat flux density ($q$): $S_q = E/q$ and heat flux ($Q$): $S_Q = E/Q = E/(q \cdot A)$.

When calculating the characteristics of the HFS, the reduced sensitivity is also used $S_V = E/(q \cdot A \cdot h)$, which does not depend on the size of the HFS and characterizes the sensitivity of a unit volume of the heat-sensitive zone. In the presented formulas, $E$ is the output signal of the HFS, $A$ and $h$ are the area and height of the heat-sensitive zone respectively.

Traditionally, the calculation of the sensitivity of HFS, depending on the design parameters and the properties of the materials used, is performed according to the formula [1, 3, 4]:

$$S_V = \frac{(\alpha_1 - \alpha_2)}{\lambda_1 \cdot f_1 \cdot (1 + \rho_{21}/f_{21}) \cdot [2 + \lambda_{21} \cdot f_{21} + \lambda_{31} \cdot (\Phi - 2 - f_{21})]}, \tag{2}$$

where $\lambda_i$ – thermal conductivity of the material,

$f_{i1} = f_i / f_1$, $\lambda_{i1} = \lambda_i / \lambda_1$, $\rho_{i1} = \rho_i / \rho_1$ – reduced cross-sectional area, thermal conductivity and specific resistance of materials, with index 3 referring to the casting compound,

$\Phi = (2f_1 + f_2 + f_3)/f_1 = 2 + f_{21} + f_{31}$ – form parameter or reduced area of a single thermoelement, for which the relation is valid:

$$\Phi = A/(Z \cdot f_1) = 1/(n \cdot f_1), \tag{3}$$

where $Z$ – the total number of thermoelements in HFS, $n$ – filling density with thermoelements.

Formula (2) was obtained under the assumption that all isothermal surfaces in the sensitive zone are planes that are parallel to the HFS surfaces and perpendicular to the incoming heat flux, i.e. there are no heat transfers between the branches of thermoelements and the area occupied by the casting compound. In fact, due to a significant difference in the thermal conductivity of materials, such flows exist, and isothermal surfaces differ from parallel planes. In practice, this leads to the fact that calculations according to formula (2) give only a general character of the dependence of the reduced sensitivity on the design parameters and material properties, and the characteristics of the manufactured sensors may differ markedly from the calculated ones.

## 4.1. Fragmentation of elementary cells of the transducer body by isothermal and adiabatic planes

In the proposed refined technique for calculating the sensitivity, the physical model of the HFS is considered as a heterogeneous body with closed inclusions with contrast thermal conductivity. The derivation of the formulas was made taking into account the recommendations [5] regarding the fragmentation of the elementary cells of the body by isothermal and adiabatic planes. This method was used in [6] to obtain calculation formulas for the thermal resistance of HFS.

The elementary cell in the HFS is a single thermoelement together with the zones of the compound related to it and, in the general case, with heat-conducting shunts and equalizing the temperature coatings, which serve to reduce the thermal resistance of HFS [6]. Figure 2 shows a single HFS element with an indication of its fragmentation by isothermal and adiabatic planes into sections, as well as equivalent circuits for connecting the thermal resistances of sections for two methods of crushing.

The first variant of fragmentation is isothermal, and the second variant is adiabatic-isothermal, since transverse heat flows equalize the temperature distribution and create isothermal surfaces.

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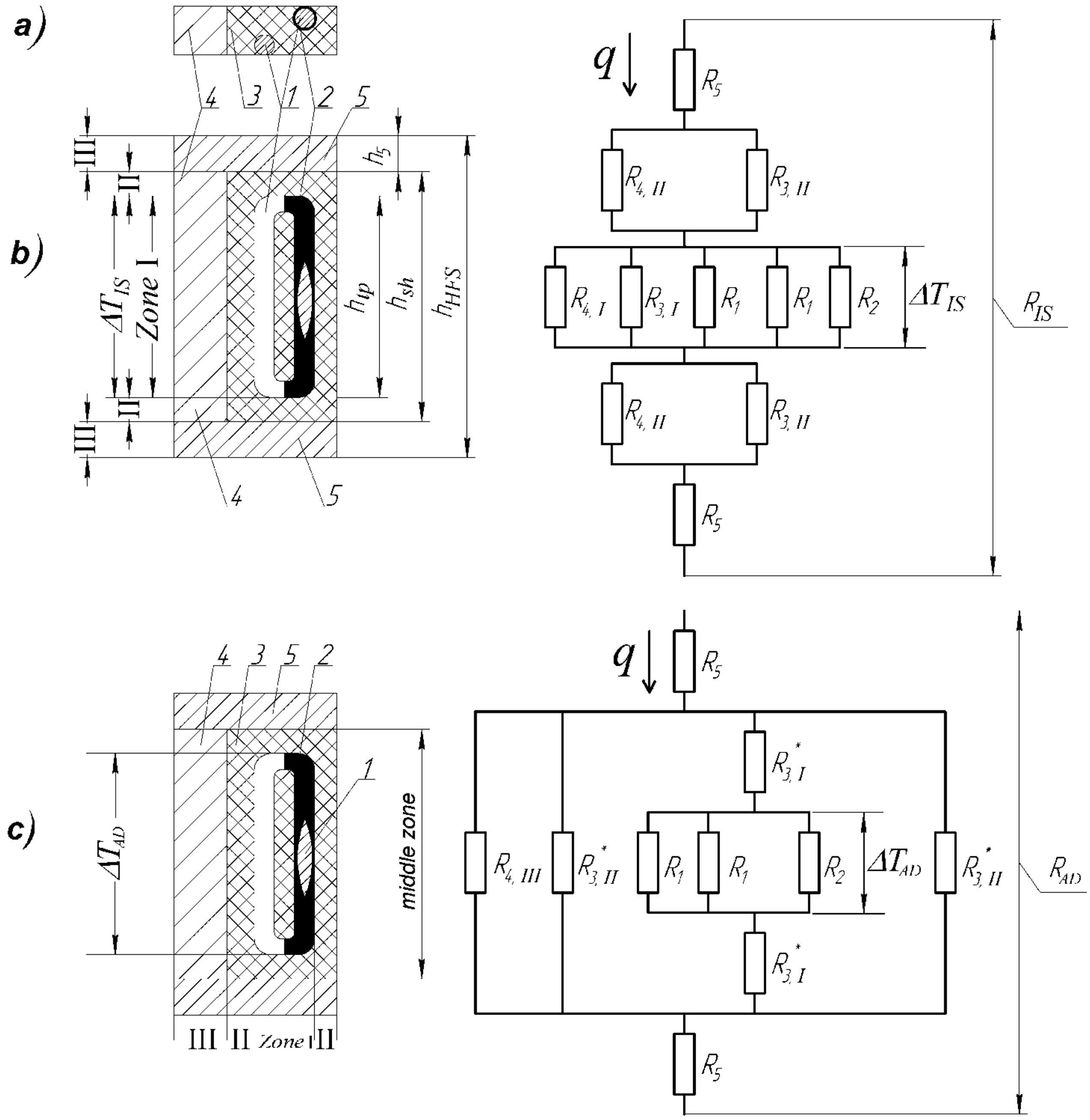


*1 – main thermoelectrode; 2 – plating; 3 – casting compound; 4 – thermal shunt; 5 – coating, equalizing the temperature.*

**Fig. 2. A single thermoelement of HFS: a) – cross-section; b) – the isothermal fragmentation and the equivalent connection circuit of the thermal resistances of the sectors; c) adiabatic-isothermal fragmentation and the equivalent circuit of the thermal resistances of the sectors.**

## 4.2. Technique for calculating the parameters of sensors

To calculate the reduced thermal resistance of each selected section $R_{sec,j}$ in the equivalent circuit, the following formula is applied:

$$R_{sec,j} = \frac{h_{sec,j}}{\lambda_i} \cdot \frac{f_{s.e.}}{f_{sec,j}} = \frac{h_{sec,j}}{\lambda_{i3} f_{i1}} \cdot \frac{\Phi}{\lambda_3}, \tag{4}$$

where $h_{sec,j}$, $f_{sec,j}$ – height and cross-sectional area of the *j-th* section; $f_{s.e.}$ – sectional area of a single element, $\Phi = (2f_1 + f_2 + f_3 + f_4)/f_1 = 2 + f_{21} + f_{31} + f_{41}$ – form parameter, $\lambda_{i3} = \lambda_i / \lambda_3$ – the reduced coefficient of thermal conductivity at $i$=1…5, and index 3 refers to the casting material, index 4 refers to the material of the shunt, index 5 refers to the material of the temperature equalizing coating.

The height of the section $h_{sec,j}$ is determined by the following parameters or their difference:

$h_{tp}$ – thermopile height, $h_{sh}$ – shunt height, $h_5$ – height (thickness) of the temperature equalizing coating, $h_{HFS}$ – HFS height, and $h_{HFS} = h_{sh} + h_5$.

Let us introduce the following notations:

$b = 2 + f_{21}$ – relative cross-sectional area of thermoelectrodes of a single element;

$\Lambda_{1,2} = 2\lambda_{13} + \lambda_{23} \cdot f_{21}$ – relative thermal conductivity per unit length of thermoelectrodes;

$\Lambda_{3,4} = f_{31} + \lambda_{43} \cdot f_{41}$ – relative thermal conductivity per unit length of passive sections of a single element (compound and shunt).

Applying Kirchhoff’s circuit laws for equivalent circuits (Fig. 2) and using the introduced notation, we obtain the values of the temperature difference between the thermoelement junctions when the heat flow/flux density acts on HFS $q$:

- for isothermal fragmentation:

$$\Delta T_{IS} = q \cdot \frac{h_{tp} \cdot \Phi}{\lambda_3} \cdot \frac{1}{\Lambda_{1,2} + \Lambda_{3,4}}, \tag{5}$$

- for adiabatic-isothermal fragmentation:

$$\Delta T_{AD} = q \cdot \frac{h_{tp} \cdot \Phi}{\lambda_3} \cdot \frac{b}{\Lambda_{1,2} \cdot (\Lambda_{3,4} + b) - \frac{h_{tp}}{h_{sh}} \cdot \Lambda_{3,4} \cdot (\Lambda_{1,2} - b)}. \tag{6}$$

Based on formulas (3), (5) and (6), we find the reduced sensitivity for the crushing options under consideration:

- for isothermal fragmentation:

$$S_{V,IS} = \frac{\alpha_1 - \alpha_2}{\lambda_3 \cdot f_1 \cdot \left(1 + \rho_{21} / f_{21}\right) \cdot \left(\Lambda_{1,2} + \Lambda_{3,4}\right)}, \tag{7}$$

- for adiabatic-isothermal fragmentation:

$$S_{V,AD} = \frac{(\alpha_1 - \alpha_2) \cdot b}{\lambda_3 \cdot f_1 \cdot \left(1 + \rho_{21} / f_{21}\right) \cdot \left[\Lambda_{1,2} \cdot (\Lambda_{3,4} + b) - \frac{h_{tp}}{h_{sh}} \cdot \Lambda_{3,4} \cdot (\Lambda_{1,2} - b)\right]}. \tag{8}$$

Note that (7), in the absence of a shunt and a temperature-leveling coating ($h_5 = 0$, $f_{41} = 0$), is equivalent to formula (2).

The refined resulting value of the reduced sensitivity of HFS is found as the average of the sensitivities for isothermal and adiabatic-isothermal fragmentation:

$$S_{V,REZ} = 0{,}5 \cdot \left(S_{V,IS} + S_{V,AD}\right). \tag{9}$$

The thermal resistance of thermoelectric HFS is calculated by the formulas:

$$R_{HFS} = 0{,}5 \cdot \left(R_{IS} + R_{AD}\right), \tag{10}$$

$$R_{IS} = \frac{2h_5}{\lambda_5} + \frac{h_{sh} \cdot \Phi}{\lambda_3 \cdot \left(\Lambda_{3,4} + b\right)} \cdot \left(1 - \frac{h_{tp}}{h_{sh}} \cdot \frac{\Lambda_{1,2} - b}{\Lambda_{1,2} + \Lambda_{3,4}}\right), \tag{11}$$

$$R_{AD} = \frac{2h_5}{\lambda_5} + \frac{h_{sh} \cdot \Phi}{\lambda_3} \cdot \frac{\Lambda_{1,2} - \frac{h_{tp}}{h_{sh}} \cdot \left(\Lambda_{1,2} - b\right)}{\Lambda_{1,2} \cdot (\Lambda_{3,4} + b) - \frac{h_{tp}}{h_{sh}} \cdot \Lambda_{3,4} \cdot (\Lambda_{1,2} - b)} \tag{12}$$

Formulas (10) – (12) are equivalent to the solutions obtained in [5], however, different parameter designations are used.

## 4.3. Experimental verification of the proposed technique

To check the correctness of the obtained relations, a number of sensors based on constantan (wire-base) with a nickel coating were specially made. Characteristics of the design of sensors: HFS diameter – 60 mm; diameter of the wire-base *d*=0,1 mm;

packing density of thermoelements $n$=3,6 mm[-2]; form parameter $\Phi\approx35$; thermopile height $h_{tp} = 1,25$ mm; HFS thickness $h_{HFS} = 1,4$ mm. The relative cross section of the nickel coating varied from $f_{21} = 0,12$ to $f_{21} = 1,1$. There were no shunts and temperature equalizing coating in the manufactured converters, therefore $h_5 = 0$; $f_{41} = 0$; $h_{HFS} = h_{sh}$; $\Lambda_{3,4} = f_{31} = \Phi - b$. When forming HFS, a compound based on the UP-610 epoxy polymer with a content of 250% filler, namely corundum powder, was used. The sensitivity of the sensors and the thermal conductivity of the compound were determined on a UVT-1 reference installation [7] in the temperature range from 323 K to 463 K. The results of calculations of the reduced sensitivity during isothermal and adiabatic-isothermal fragmentation, as well as the refined average value in accordance with formulas (7) – (9) and experimental studies, are presented in Figure 3 in the form of graphs of the dependences of the reduced sensitivity on the relative cross section of the nickel coating $f_{21}$. All results refer to an average HFS temperature of 323 K. As follows from the graphs, the results of the experimentally measured reduced sensitivity are in good agreement with the values obtained in accordance with the proposed calculation technique.

The values of thermal conductivity, thermoelectric coefficients and resistivity of materials depend significantly on temperature, as a result of which the reduced sensitivity also has temperature dependence. Figure 4 shows the graphs of the temperature dependences of the reduced sensitivity of HFS for three values of the relative cross section of the nickel plating, obtained by the proposed refined method, taking into account the temperature dependences of the properties of the materials. The same figure shows the experimentally obtained values of the sensitivity of the corresponding sensors.

The experimental data are in good agreement with the calculation results, and the sensitivity of the sensor with $f_{21} = 1,1$ is practically independent of temperature, which indicates the possibility of creating HFS sensitive to heat flux, but invariant to temperature change.

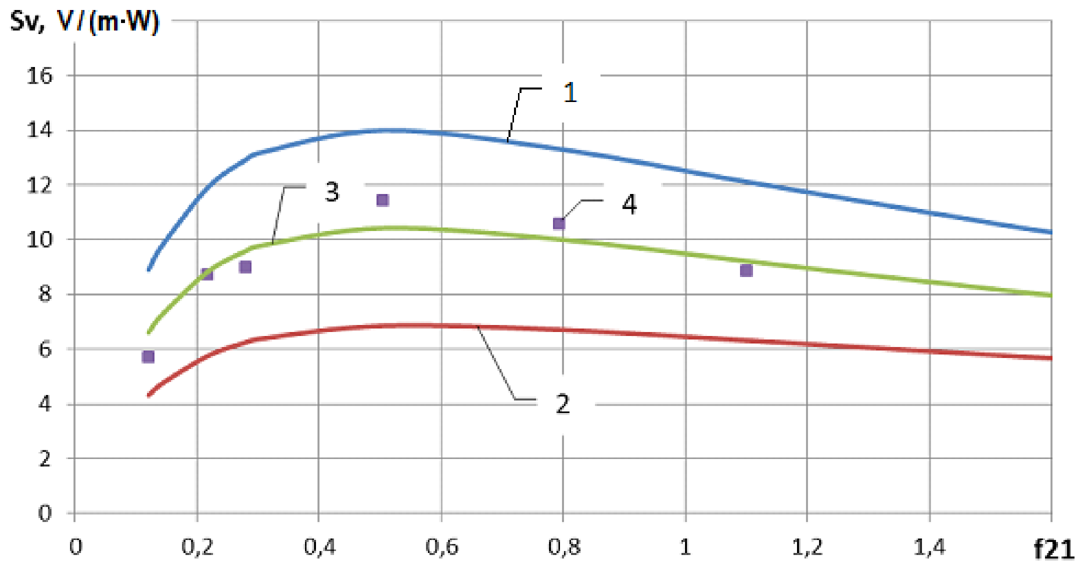


**Fig. 3. Dependences of the reduced sensitivity on the relative section of the nickel coating $f_{21}$: 1 – for isothermal fragmentation $S_{V,IS}$; 2 – for adiabatic-isothermal fragmentation $S_{V,AD}$; 3 – resulting value $S_{V,RES}$; 4 – experimental data $S_{V,EXP}$.**

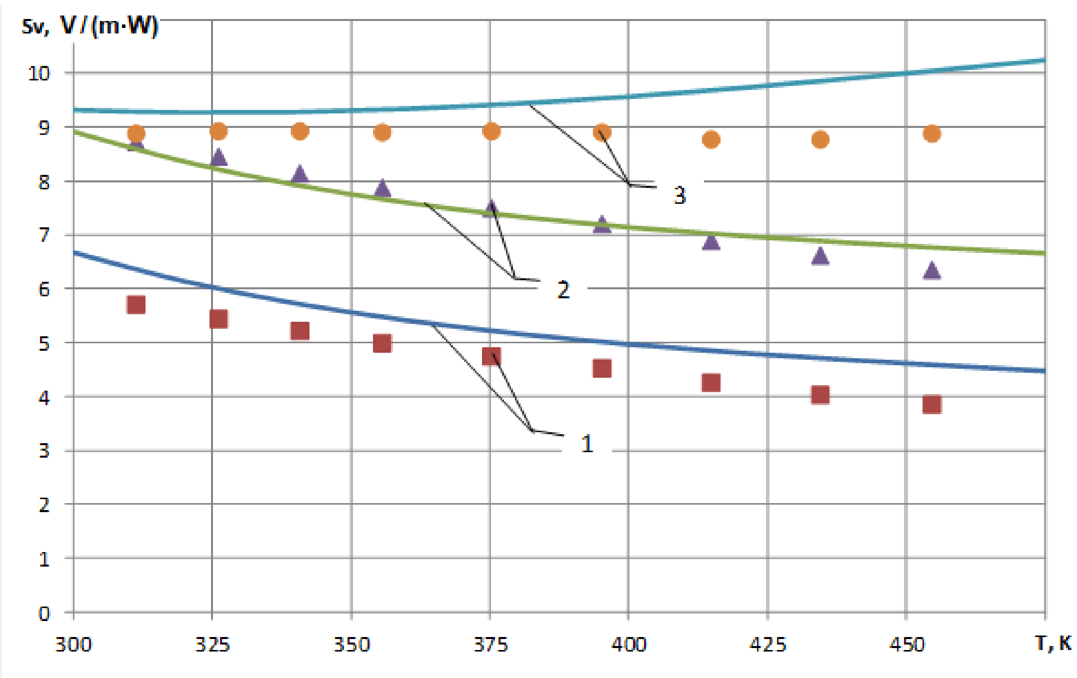


**Fig. 4. Calculated (solid lines) and experimental (points) temperature dependences of the reduced sensitivity of HFS for three values of the relative cross section of the nickel plating: 1 – $f_{21}$=0,12; 2 – $f_{21}$=0,217; 3 – $f_{21}$=1,1.**


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### 4.4. Analysis of the influence of the thickness of the protective layer of the compound on the sensitivity of the sensor

In practice, it was noticed that the small thickness of the protective layer of the compound separating the thermopile junctions from the HFS surface affects its sensitivity. This phenomenon can adversely affect the accuracy of measurements in the case when the sensor is calibrated and operated under different heat transfer conditions, for example, calibration occurs with the conductive method of heat supply, and operation occurs with the radiative-convective method. In [8], to stabilize the sensitivity of HFS, it is recommended to make a protective layer with a thickness of 0.1…0.5 mm. At the same time, the traditional formula (2) does not demonstrate the dependence of sensitivity on the thickness of the protective layer of the compound. In the proposed refined calculation procedure, this dependence is taken into account by the presence in the denominator of formula (8) of a factor that is equal to the ratio of the thermopile and shunt height (or HFS). Figure 5 shows the dependences of the reduced sensitivity of HFS without shunts and temperature equalizing plates on ratio $h_{HFS}/h_{tp}$ with variation of the form parameter $\Phi$ for constantan-nickel and constantan-copper thermopiles.

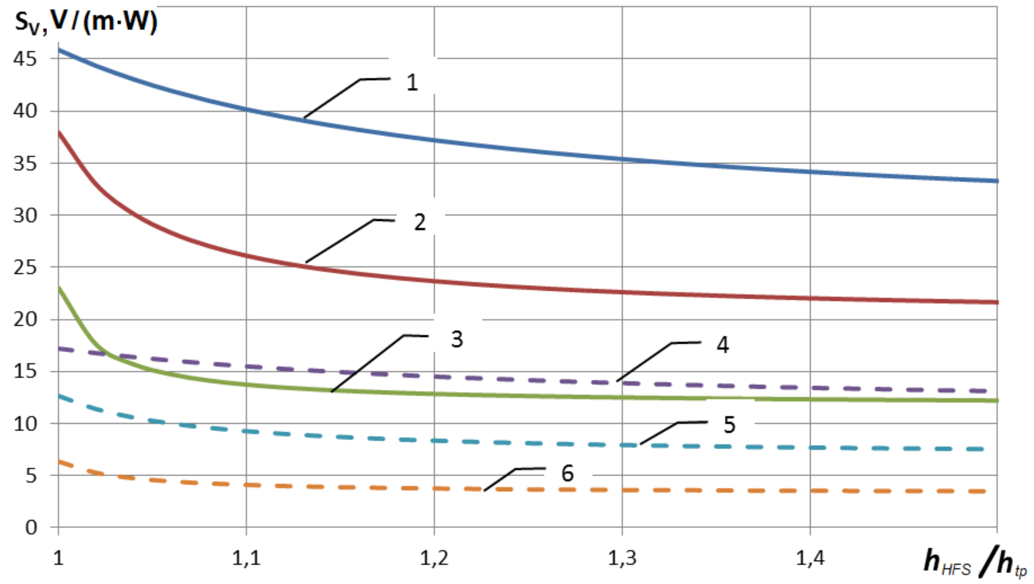


**Fig. 5. Dependences of the reduced sensitivity on the ratio $h_{HFS}/h_{tp}$ when varying the shapes of the parameter Φ for constantan-copper (solid line) and constantan-nickel (dashed line) thermopiles: 1 – Con-Cu, Φ=10; 2 – Con-Cu, Φ=50; 3 – Con-Cu, Φ=200; 4 – Con-Ni, Φ=10; 5 – Con-Ni, Φ=50; 6 – Con-Ni, Φ=200.**

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The graphs show that in the range of values of the ratio $h_{HFS}/h_{tp}$ from 1.0 to 1.2, the sensitivity significantly depends on the thickness of the protective layer, and at $h_{HFS}/h_{tp} \geq 1{,}2$ the value of the reduced sensitivity stabilizes, and a further increase in the thickness of the protective layer has little effect on it. Thus, it is really possible to recommend for the most common sensors with a height of $h_{HFS} \approx (1...4)$ mm, to make protective layers on both sides of the HFS with a thickness of 0.1...0.4 mm.

**Conclusions**

Using the fragmentation method of a single HFS element by isothermal and adiabatic planes, new refined formulas have been obtained for calculating the reduced sensitivity of bimetallic galvanic HFSs of the form of an additional wall. Experimental studies of a batch of constantan-nickel sensors with different sections of electroplated coating confirm the reliability of the proposed calculation formulas. The results obtained theoretically confirm the well-known fact of the dependence of sensitivity on the thickness of the protective layer of the compound and allow to estimate the necessary value of this thickness for stabilizing sensor characteristics.