KAPITEL 7 / CHAPTER 7 [7]
LOCALIZATION OF DAMAGE REGIONS IN LAMINAR COMPOSITES USING CONTINUOUS WAVELET TRANSFORMS

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Introduction

The widespread use of laminar composites in a variety of industrial applications necessitates local and integral non-destructive quality control and safe operation of composite-based structures [1]. Currently, a significant portion of the structural elements of vehicles, architecture and light industry include fiber-reinforced multilayer composites. Factors such as long-term use, chemical degradation and mechanical impact can lead to four main types of damage, namely delamination, matrix cracking, fiber rupture and interface delamination. As a rule, difficulties in the operation of composite systems arise due to a combination of these damages. Early detection of initial damage can prevent catastrophic failure or structural deterioration beyond repair.

Therefore, it is critical to detect and analyze these damages for practical materials or structures. Due to the anisotropy and complexity of the internal structure, non-destructive testing for damage detection on composites is more difficult than on metals. Traditional non-destructive testing methods such as acoustic scanning, X-ray and eddy current require expensive equipment and strictly controlled environmental conditions.

Currently, damage detection in local volumes of laminated composites is carried out based on non-destructive testing of vibration oscillations [2]. This technique is most effective for active damage detection, i.e. for creating excitation in the structure, which does not require external equipment The technique of damage detection of materials requires monitoring of such quantities as dynamic response, modal coefficients and spectral bandwidth. The basic idea of vibration-based damage detection is that these parameters depend on the physical properties of the structure being inspected. Therefore, changes in the physical properties of the structure due to damage can lead to detectable changes in its parameters, such as natural frequencies, displacement or deformation modes and modal attenuation Most vibration signals include a wide range

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of wave numbers and the main problem of analysis is to extract useful information for damage detection or identification. Fast spectral transform is one of the most widely used and well-established methods among many other signal analysis methods. Critical damage usually occurs as a result of the development or accumulation of a large population of relatively small damage. Therefore, the problem of detecting damage at an early stage of its development is of primary importance. However, when damage is very small, the damage-induced changes in the physical properties of composites are always too small to detect damage using a fast spectral transform-based method. In addition, the measured vibration signals are often contaminated with noise. Considering all the above, it can be stated that the wavelet transform-based method for vibration signal analysis is gradually accepted in many fields. One of the main reasons is its good localization in time and frequency.

The technique of using different variations of 2-D wavelet transforms is effective for damage detection in laminated composites [3]. In particular, it was found that discrete wavelet transforms using B-spline wavelets for these types of composites have an advantage over other wavelet transform methods. The vibration mode shapes of sandwich composite structures are determined based on the analysis of impact damage, which is most often encountered during the manufacturing and maintenance of the structure.

Localization of local damage areas in individual layers of composite structures requires the use of fractional B-spline wavelet techniques [4]. It should be noted that experimental studies have shown that the use of inseparable quincunx wavelets on one hand and ultrasonic scanning of the composite volume on the other hand lead to virtually indistinguishable results in localizing macroscale damage. In addition, the method of analyzing the squared curvature of the vibration mode shape, supplemented by the results of continuous wavelet transform, is quite often used to detect and localize damage in ribbon-type composites.

However, the number of publications describing the features of its application to vibration-based damage detection of composites is not large enough. This problem, despite the extensive research on vibration analysis of damaged laminated composites, is still not fully understood. Only a few effective and practical methods have been developed for the early detection and size determination of damage such as internal delamination. Therefore, this paper focuses on the further development of a method for the effective detection of internal delamination in fiber laminated plates. The characteristic feature of this method is the combination of numerical analysis of structural modal parameters with wavelet packet decomposition of vibration signals.

7.1. Model of local deformations

The structural deformation method is used to calculate the modal parameters such as natural frequency, mode shape, and modal strain of each mode for laminated composite plate with or without delamination. For the numerical analysis of the vibration response of delaminated composite materials, an effective structural deformation model was used to accurately estimate the material anisotropy and delamination parameters. The limitations of the model developed in this study concern the small thickness of the composite specimens. For laminar composites, a three-dimensional local strain model is adopted, which is characteristic of the combined effects of transverse shear stress on the performance of the composite volume.

The finite element used for the dynamic analysis of the behavior of a multilayer composite plate is a rectangular thin plate with a finite number of nodes. The displacements along the global coordinate axes x, y, and z determine the number of degrees of freedom for each node. It is assumed that the element thickness and the thickness of the corresponding individual plate are equal. The local coordinate system of the element (x_1; y_1; z_1) is organized so that the first axis coincides with the direction of the fibers. All mechanical parameters are assumed to be the same over the local volume of the entire element. For a local volume with a finite number n of nodes, each of which is characterized by three degrees of freedom, the displacement field along the element is defined as

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$$\{\delta\} = (u,\upsilon,w)^T = \sum_{i=1}^{n} [N_i]\{\delta_i\}, \tag{1}$$

where $\{\delta_i\} = (u, \upsilon, w)^T$ is the displacement vector at node i; $[N_i] = N_i\,[I_i]$ is the three-order unit matrix and Ni is the shape function.

In this case, the deformation vector of each element is functionally related to the displacement vector in the global coordinate system

$$\{\varepsilon^e\} = (\varepsilon_{11}^e, \varepsilon_{22}^e, \varepsilon_{33}^e, \varepsilon_{12}^e, \varepsilon_{13}^e, \varepsilon_{23}^e)^T = \sum_{i=1}^{n} [B_i]\{\delta_i\}, \tag{2}$$

where $[B_i] = [\Delta]\,[N_i]$ and Δ is the orthogonal operator.

The strain vector in the local volume of the laminar composite can be expressed through nodal displacements as

$$\{\varepsilon^e\} = [B\}\{\delta^e\} \tag{3}$$

with

$$[B] = [[B_1],[B_2],...,[B_n]] \tag{4}$$

and

$$\{\delta^e\} = \left(\{\delta_1\}^T, \{\delta_2\}^T, ..., \{\delta_n\}^T\right)^T. \tag{5}$$

Thus, the mechanical stress of a local volume element in the global coordinate system can be expressed through nodal displacements as

$$\{\sigma^e\} = (\sigma_{11}^e, \sigma_{22}^e, \sigma_{33}^e, \sigma_{12}^e, \sigma_{13}^e, \sigma_{23}^e)^T = [K^e]\{\delta^e\}, \tag{6}$$

where $[K^e]$ is the element stiffness matrix as

$$[K^e] = \int_{V_e} [B]^T [A][C][A]^{-1}[B]dV. \tag{7}$$

The next step of the calculation model of local stresses in a laminated composite is the summation of the nodal displacements of all elements. In this case, the total deformation energy of a multilayer composite plate can be represented as

$$U = \frac{1}{2}\{\delta\}^T [K]\{\delta\}, \tag{8}$$

where $\{\delta\}$ and $[K]$ are the global nodal displacement vector and stiffness matrix, respectively. Solution of the eigenvalue problem

$$\left([K]-\omega^2[M]\right)\{\delta\}=0 \tag{9}$$

allows to determine modal parameters such as natural frequencies ω_i, mode shapes, modal deformations, etc.

The case of an arbitrary solid laminated composite plate is considered separately when describing damage in laminar composites. In order to ensure continuity of the material in the volume of the sample, the displacements and their variations of each pair of coinciding nodes on the upper and lower adjacent plates must be equal throughout the calculation process. The case of delamination of the laminar composite in the calculation model corresponds to the absence of a connection between the displacement of each pair of coinciding nodes on the upper and lower surfaces in the delamination region.

One of the new algorithms for signal processing and data compression is the wavelet transform. This transform is used to analyze signals with multiple resolutions. Compactly supported wavelets can be extended to the domain of wavelet packets. If φ(t) is any sufficiently good function with mean 0 and defining its modulation, spreading and translation as $e^{ift}\varphi(t)$, $s^{1/2}\varphi(s\cdot t)$(and $\varphi\,(t - p)$, respectively, one can define a set of modulated, spread and translated wavelets, which in turn form a family of wavelet packets with parameters f, s and p.

Energy conservation is performed during these transformations. The possibility of normalizing waves as single packets is due to the conservation of energy during the propagation of signals through the volume of the composite material. The component of the function x at f, s and p is the inner product of x with the modulated waveform, whose parameters are f, s and p. For the case of a large value of the normalized component of the function, the value of x has a significant energy level near the frequency f, scale s and position p. Therefore, the decomposition of the wavelet packet is effectively used for the analysis of the vibration response, especially for non-stationary signals.

A single structured wavelet packet is a square integral modulated wave with mean value 0. This packet is well localized both in position and frequency. Because of this, it is possible to assign three parameters to a wavelet packet, namely frequency, scale, and position. If ψ is a wavelet packet, then the frequency and position can be taken as the centers of mass $|\psi|^2$ and $|F(\psi)|^2$, where $F(\psi)$ is the Fourier transform of ψ. The characteristic width jwj can be considered as the unit of scale. Typically, the exact quadrature mirror filter $h(k)$ can be written as

$$\sum_k h(k-2i)h(k-2j)=\delta_{ij} . \qquad (10)$$

The orthonormal scaling function $\varphi(t)$ and the wavelet function $\psi(t)$ satisfy the dynamic scaling equation in multiscale analysis

$$\varphi(t)=\sqrt{2}\sum_k h(k)\varphi(2t-k) , \qquad (11)$$

$$\psi(t)=\sqrt{2}\sum_k g(k)\varphi(2t-k) , \qquad (12)$$

where $g(k) = h\,(k+1)(-1)^k$.

The binary scaling equations given above are constrained by the conditions $w_0 = \varphi(t)$, $w_1 = \psi(t)$ and can be expressed as recursive equations

$$w_{2n}(t)=\sqrt{2}\sum_k h(k)w_n(2t-k) , \qquad (13)$$

$$w_{2n+1}(t)=\sqrt{2}\sum_k g(k)w_n(2t-k) . \qquad (14)$$

Thus, the spectrum of wave modes could be renormalized in terms of the width of the characteristic deformation band.

7.2. Decomposition of functions into an orthonormal wavelet packet

Decomposition operators for multi-analysis of the deformation field are defined by the following relations

$$H[s_k](i)=2\sum_k s_k h(k-2i) , \qquad (15)$$

$$G[s_k](i)=2\sum_k s_k g(k-2i) . \qquad (16)$$

In this case, the boundary conditions of the first and second types can be rewritten as follows

$$w_{2n}(t-l)=H[w_n(2t-k)](l), \tag{17}$$

$$w_{2n+1}(t-l)=G[w_n(2t-k)](l) \tag{18}$$

or, after unification

$$w_n(t-l)=\sqrt{2}\sum_k h(l-2k)\,w_{2n}\left(\frac{t}{2}-k\right)+g(l-2k)w_{2n+1}\left(\frac{t}{2}-k\right). \tag{19}$$

If for the characteristic function of propagation of a wave packet in the volume of a laminated composite the following constraint can be written

$$f(t)=\sum_k s_k^j w_n\left(2^{-j-1}t-k\right), \tag{20}$$

then its binary decomposition of j layers within a fixed orthonormal wavelet packet can be obtained as a solution to the following equation

$$f(t)=\sqrt{2}\sum_i H[s_k^j](i)\,w_{2n}\left(2^{-j-1}t-i\right)+\sqrt{2}\sum_i G[s_k^j](i)\,w_{2n+1}\left(2^{-j-1}t-i\right). \tag{21}$$

Thus, using the wavelet packet decomposition, any signal that propagates in a laminated composite and is modulated by a local strain field can be decomposed into two parts. The first component is ortho-normalized to *H*, and the second component is determined by the value of *G*.

It is obvious that the wavelet packet decomposition of the signal has a better localization effect than the wavelet, and is therefore used to adaptively select the appropriate frequency band according to the signal characteristics. In turn, the signal characteristics are determined by the volumetric strain density. Improving the identification of local strains in composites requires increasing the resolution in both the frequency and time domains.

At the early stage of deformation initiation, the differences in signals between intact and damaged structures are usually insignificant. In this case, extracting the damage index directly from the measured signal shape (even decomposed into wavelet packets) is still difficult. Therefore, energy spectrum analysis is used to increase the sensitivity of features to damage. The second-order norm of the original signal $f(t)$ is

$$\|f\|_2^2=\int_R |f(x)|^2\,dx. \tag{22}$$

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Under these conditions, equation (22) can be considered as the equivalent energy of the original signal in the time domain. For a wavelet *w* resolved in the continuous spectrum, we have

$$\iint_R \left| W_\varphi f(a,b)/a \right|^2 dadb = \|f\|_2^2 . \tag{23}$$

Equation (23) illustrates the equivalent relationship between the energy of the wavelet transform and the energy of the original signal. Thus, the developed model allows us to reliably express the change in energy in the original signal by the energy spectrum of the response signal decomposed into wavelet packets.

It should be noted that the sum of the squares of the decomposed signal in the energy spectrum of wavelet packets is chosen as the energy characteristic in each subspace (frequency range). In the subspace V_{2i}, i.e. the *i*-th frequency range of the *j*-th layer, the result of the wavelet packet decomposition is expressed as $\{S_i(k), k = 1, 2, \dots, M\}$, and its energy is expressed as

$$U_{2i} = \sum_{k=1}^{M} \left| S_i(k) \right|^2 , \tag{24}$$

where *M* is the length of samples in the subspace.

For the case when the values of $\{U_{20}\}$ and $\{U_{2d}\}$ can be compared with the energy spectra of signals measured from intact and damaged laminated composites, respectively, then the dimensionless index is equal to

$$\eta_i = \frac{\left| U_{20} - U_{2d} \right|}{U_{20}} \tag{25}$$

can be correlated with the change in energy of the damaged area of the composite in subspace V_{2i}. In addition, it should be noted that the frequency band of the vibration signal, in accordance with the restrictions imposed on the calculation model, must be the same for each subspace obtained by decomposing the eigenvalues of the local deformation field of the laminated composite.

The initial occurrence of deformation in a composite material or structure usually leads to an increase in damping, which is associated with energy dissipation during vibration processes. The dominant energy dissipation in laminated fiber composites occurs not only due to the viscoelastic nature of the matrix and/or fiber materials, but also due to interfacial damping.

The propagation of non-monochromatic waves in the composite volume is accompanied by an increase in local energy dissipation in the damaged area. Analysis of the calculation model results for both local and global values of the energy dissipation gradient due to delamination showed that the speed and direction of acoustic waves directly affect the dynamic characteristics of energy dissipation and the characteristic dimensions of delamination.

The results of numerical calculations for the localization of deformation regions in laminated composites were based on the geometric and physical characteristics of real samples of composite material. The calculations were performed using data from five rectangular specimens made of multilayer carbon fiber reinforced epoxy composites. All specimens had an equal area of 320 × 210 mm^2 and a number of layers (N_5 = 18) with orientation $[0^0/0^0/90^0/90^0/0^0/0^0/90^0/90^0]_s$. In turn, the thickness of each laminate layer was 0.11 mm. The specimens were numbered using indices: Sm_k (k = 1 – 5). The location of the deformation regions varied for different specimens. Modeling of the delamination process was based on multiple arrangement of 0.015 mm thick polyester film in the specimen.

Each specimen was characterized by a rectangular delamination, the location of which was described by a coordinate axis directed from the center of the specimen to its edge. The delamination areas of specimens Sm_1, Sm_2, Sm_3, Sm_4 and Sm_5 are 16, 31, 54, 61 and 67 mm^2, respectively. The mechanical constants of the composite material for the calculated specimens are E_1 = 125 GPa, E_2 = E_3 = 6.1 GPa, E_4 = 22 GPa, E_5 = 34 GPa, G_{12} = G_{13} = 4.80 GPa, G_{23} = 3.11 GPa, G_{35} = 2.85 GPa, G_{42} = 1.74 GPa, υ_{12} = υ_{13} = υ_{23} = 0.8, and ρ = 2400 kg/m^3.

The calculation results for the natural frequencies for the first six modes of samples with indices Sm of composites with different boundary conditions (B_I and B_{II}) are given in Tables 1 – 2.

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Table 1 - Frequencies (Hz) with modes *k* = 1 - 8 for composite plates (B_I)

k	Sm_1	Sm_2	Sm_3	Sm_4	Sm_5
1	99.24	97.18	97.12	96.31	95.64
2	200.91	197.74	195.23	194.02	191.04
3	303.23	301.90	294.63	285.86	273.70
4	410.74	409.21	456.47	425.71	451.08
5	506,99	501.34	487.08	462.20	426.79
6	611.61	605,16	582.97	574.91	563.02
7	714.07	701.19	692.35	685.05	674.21
8	820.64	808.11	786.12	761.18	747.24

Table 2 – Frequencies (Hz) with modes *k* = 1 - 8 for composite plates (B_{II})

k	Sm_1	Sm_2	Sm_3	Sm_4	Sm_5
1	152.80	150.40	149.70	147.50	145.80
2	243.08	234.29	225.25	217.48	209.30
3	330.37	325.61	317.31	305.65	294.65
4	423.94	420.22	411.45	397.51	382.63
5	509.19	495.53	476.33	468.86	454.98
6	598.54	586.32	57184	563.63	552.40
7	684.72	678.35	663.98	651.27	642.45
8	771.00	768.45	753.03	744.46	721.63

It can be seen that with the increase of the delamination area, the natural frequency decreases for each fixed mode. In turn, within fixed area for each laminated composite sample, a nonlinear increase in mode frequency is observed. The change in natural frequency caused by delamination, however, is very small and almost impossible to measure.

The relative displacements d in the x–y plane for the modes $M\ i,k$ ($i = 1 – 6$; $k = u, b$) on the upper (u) and lower (b) surfaces within the laminated region of the plate for the boundary condition B_I are shown in Figure 1.

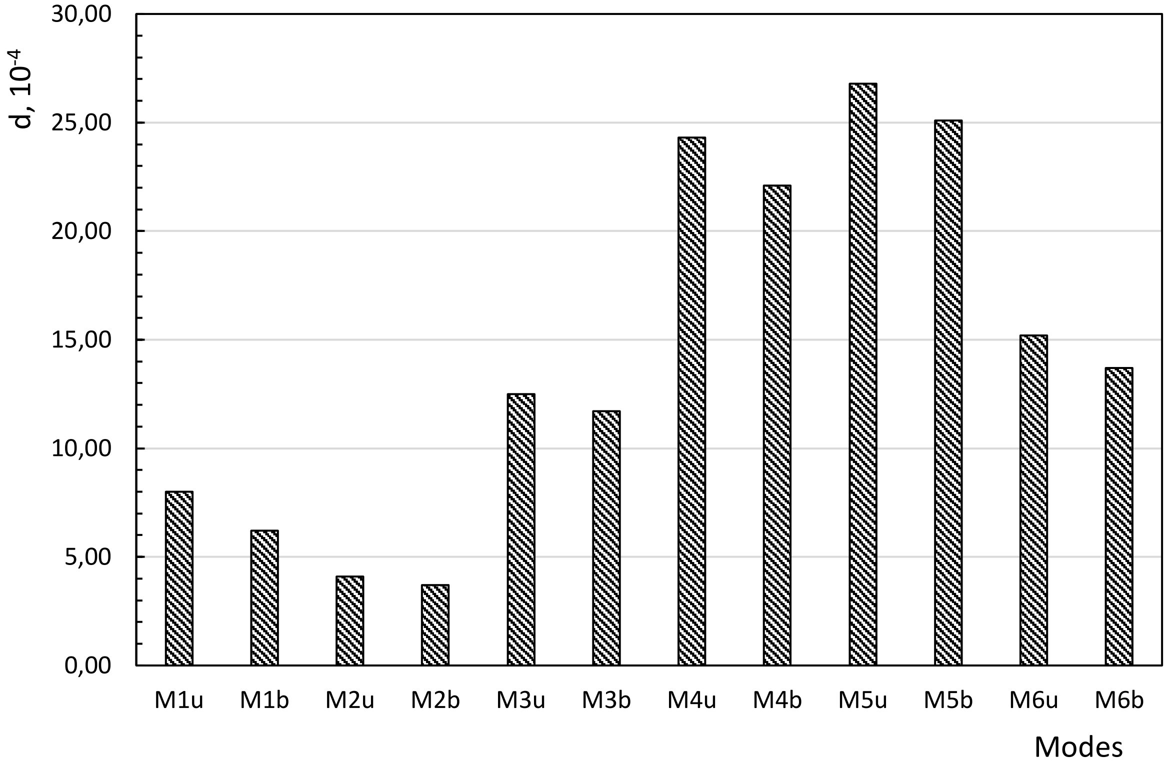

Figure 1 – Relative displacements in the x–y plane

Summary and conclusions.

Multilayer laminated composite specimens with a large degree of delamination were analyzed both numerically and experimentally. The change in physical properties caused by the effects of delamination and the occurrence of a deformation field were investigated by analyzing the modal parameters such as natural frequencies and mode shapes. The response signal from the interaction of a wave with a local deformation was processed using wavelet packet decomposition to extract the change in energy dissipation. It was found that the change in frequency and mode shape at small deformations is quite small. It is proved that the change in physical properties is mode dependent and is a result of the change in energy dissipation due to the impact and interactive motion in the delamination region. Based on the experimental data on delamination inside the laminated composites, the spatial distribution of the mode