KAPITEL 5 / CHAPTER 5 [5]
DAMAGE IDENTIFICATION IN ANISOTROPIC MEDIA

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Introduction

A large number of both experimental and theoretical works have been devoted to the elastic wave-based structural monitoring technology in recent years. Lamb waves, which are used for such techniques, have several advantages. Such wave packets propagate over long distances, have high sensitivity to internal defects, and also allow interrogation through the thickness of the composite sample. Lamb wave can be used as an effective mechanism for interrogating plate structures for the purpose of damage identification. In particular, the methods use the characteristics of the conjugate finite element and normal expansion. Such characteristics are optimal for studying the in-phase and phase-delayed excitation of the Lamb wave [1 – 5].

The experimental implementation of phase delay is realized using multiple transducers integrated with composite laminates by considering coupled or embedded transducers. From the point of view of mechanical damage identification, the Lamb wave propagation in plates integrated with piezoelectric sensors/actuators is of great interest. The analysis of the wave packet propagation features is based on the Mindlin plate theory. The propagation of Lamb wave packets in laminar composite structures is characterized by transverse shear and rotational inertia in the host plates. The practical implementation of the generation and propagation of the A0 Lamb wave mode was achieved by symmetrically installing disk actuators of piezoelectric sensors on the side surface of laminated composite samples [6]. In this case, the piezoelectric effect of the disks was the cause of the equivalent bending moment.

The extension and generalization of the conjugate finite element and normal mechanical displacement method is based on the optimal generation of the A0 Lamb wave mode in a composite plate. The generation of wave packets with symmetric and antisymmetric wave modes is associated with the use of surface-coupled piezoceramic

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transducers, which can be used at subsequent stages to detect mechanical damage. Lamb wave propagation in composite plates is a complex phenomenon dependent on a large number of parameters. Typical examples of such parameters are multimode characteristics, dispersion coefficients, and wave velocity versus direction. The addition of stiffeners to a structure containing laminar composite inserts can also introduce additional complexity.

The study of Lamb wave mode transmission and mode conversion through adhesive bonds in laminar composites can be investigated quite effectively using the finite element method [7]. A number of separate studies have been aimed at analyzing the propagation of Lamb waves in laminar composite samples of variable thickness. In this case, the hybrid boundary element method for discrete mode transformations has proven to be the most promising. Despite these studies, it is still worth recognizing that the main successes in identifying mechanical damage have been achieved for homogeneous and isotropic plates of laminar composite structures with stiffeners. A non-destructive testing method based on Lamb waves is proposed in this paper for damage detection in reinforced composite panels. Combining anisotropic wavefront and time-of-flight algorithm, the method utilizes the properties of Lamb waves to quantify delamination and impact damage. Numerical simulations and experimental tests are used on reinforced composite panels with cross-laminated composite laminates to verify the proposed method.

5.1. Method of symmetric and anti-symmetric modes of Lamb waves

In general, the Lamb wave approach to damage detection is characterized by (the ability to inspect large structures while preserving coating and insulation. In addition, the ability to inspect the entire cross-sectional area of the structure is preserved. The Lamb wave packet-based technique has high sensitivity to multiple defects with high identification accuracy. The analysis of Lamb wave propagation in anisotropic viscoelastic media is quite a challenging task.

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With very high speed, waves reflected from boundaries can easily hide components scattered by damage in signals. To ensure accuracy, the structure to be tested can be relatively large and with a relatively small detection area. Lamb waves are usually characterized by several wave modes. The dispersion properties of such wave formations are not identical throughout the thickness of the composite, even for the same mode, but in different frequency ranges. Existing methods of both experimental and theoretical studies provide the possibility of identifying damage using Lamb waves for fiber-reinforced composite structures. Lamb waves propagating in composite structures have unique characteristics of dispersion processes. The features of Lamb wave propagation in composite laminates provide a free choice of their generation mode.

Lamb waves, consisting of a superposition of longitudinal and shear modes, are observed in relatively thin laminated composite plates. Their propagation characteristics vary with the angle of entry, excitation, and structural geometry. Lamb waves, consisting of a superposition of longitudinal and shear modes, are observed in relatively thin laminated composite plates. Their propagation characteristics vary with the angle of entry, excitation, and structural geometry. A Lamb mode can be either symmetric or antisymmetric and are described, respectively, by the following relations:

symmetric modes

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2qp}{\left(k^2-q^2\right)^2}, \quad (1)$$

anti-symmetric modes

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{\left(k^2-q^2\right)^2}{4k^2qp}, \quad (2)$$

$$p^2 = \frac{\omega^2}{c_L^2} - k^2, \quad q^2 = \frac{\omega^2}{c_T^2} - k^2, \quad k = \frac{\omega}{c_p}, \quad (3)$$

where
h is the plate thickness;
k is the wavenumber;

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c_L is the velocity of longitudinal mode;
c_T is the velocity of transverse mode;
c_p is the velocity of phase mode;
ω is the circular frequency of Lamb wave.

The relationship between propagation speed and frequency implies that Lamb waves, regardless of mode, are dispersive (speed depends on frequency). The presence of Lamb wave modes is accompanied by a transverse (shear) motion different from normal shear waves (vertical shear mode) in laminated composites. Perpendicular to the plane of the composite cross-section of propagation, such a mode has been accordingly called the shear horizontal (SH) mode (Lamb wave).

The anisotropic properties of composite structures give rise to physical processes such as direction-dependent velocity and the difference between phase and group velocities. In an *N*-layer composite laminate, the Lamb wave can generally be described using its displacement field *u*, satisfying the Navier displacement equations in each layer

$$\mu^n \nabla^2 u^n + \left(\lambda^n + \mu^n\right) \nabla(\nabla u^n) = \rho^n \frac{\partial^2 u}{\partial t^2}, \quad n = 1, 2, \ldots, N \tag{4}$$

where
ρ^i is the density of the *i*th layer;
λ^i is the first Lame’ constan of the *i*th layer;
μ^i is the second Lame’ constant of the *i*th layer.

In most cases, when propagating through the volume of a composite, there is attenuation in magnitude, a change in the propagation speed, and a change in wave number, called dispersion.

Analysis of experimental measurements shows that Lamb waves are capable of propagating over relatively large distances even in composites. Larger propagation distances are typically observed in carbon fiber-based materials than in glass fiber-reinforced materials. Artificial or natural stiffening can slightly increase attenuation. The most serious influence on attenuation is exerted by the presence of surface coating materials, which can cause very significant attenuation.

Accurate consideration of the boundary conditions on each layer of the laminar composite leads to a complex dispersion equation

$$\left|A\left(\omega, k, \lambda^{n}, \mu^{n}, h_{n}\right)\right| = 0, \quad (5)$$

where the conditional dependence can be considered: $\omega = \omega\,(k, h_n, \lambda_n, \mu_n)$.

Lamb wave fault identification essentially depends on the interpretation of the captured wave signals. Determining the signatures useful for fault identification from the collected Lamb wave signal typically involves a number of interferences. Such interferences include: contamination by various noises, interference from natural structural vibration, confusion of several modes, and bulkiness of the sample data. In this regard, various signal processing and identification methods have become widespread, in particular, time series analysis, frequency analysis, and integrated time-frequency analysis.

Time domain signal analysis can detect damage both globally and locally. In particular, delamination in a composite beam can be detected by measuring the time of flight in the final Lamb signal. Time series analysis can be applied to waveforms for damage detection using a two-stage prediction model. As a result of applying this technique, it can be found that the difference in time domain signals between the defective structure and the reference, defined as the residual error, will be greatest for sensors near the damage.

A slightly different approach to structural damage detection is based on the combination of independent component analysis in the time domain. This technique allows the detection of key features from the measured vibration signals. However, with the exception of a few successful applications in fault localization, direct time series analysis is usually unable to isolate information scattered across defects properly from noise in different frequency ranges. In addition, a reference signal is needed for comparison.

A significant number of works are devoted to the study of a dynamic signal in the frequency domain using the Fourier transform. This transform mathematically transforms the time-dependent Lamb wave signal, f(t), into frequency space according to the equation

$$F(\omega)=\int_{-\infty}^{\infty} f(t)\exp(-j\omega t)dt \quad . (6)$$

where ω is the angular frequency;
j is the unit complex;
F(ω) is the Fourier counterpart of f(t).

The fast Fourier transform and its two-dimensional form, which are derived from the Fourier transform but with improved capabilities, are often used to analyze Lamb wave signals, and can significantly speed up the calculation process. The simplification of the explicit analysis of multimode Lamb waves can be obtained by measuring the response of a composite plate at a number of equally spaced positions on the surface and then applying a 2D Fourier transform. In subsequent steps, different modes at different frequencies in the frequency-wavenumber domain can be distinguished.

As an example of an experimental setup, a laser ultrasound system for generating Lamb waves in the direction perpendicular to the laser beam can be mentioned. In this case, the spatial orientation of the laser line source was controlled by mirror translation. Subsequently, a two-dimensional Fourier transform was applied to the signals collected from different positions using a Michelson interferometer along the scanning path. In this way, dispersion curves could be obtained.

A similar principle was used as the basis for the experimental setup, where a pair of transducers was used to measure signals at equally spaced positions. The second stage involved analyzing the two-dimensional Fourier transform. This analysis ended with the separation of symmetric and antisymmetric Lamb modes. The resulting Lamb wave spectrogram in the frequency-wave number region contained a region where several modes were already separated, even for those in the same frequency band.

The deficiencies of dynamic Lamb wave analysis in the time or frequency domain can be addressed by introducing a packet that combines time information with frequency data. Most time-frequency algorithms can be summarized as follows

$$P(t,\omega)=\frac{1}{2\pi}\int\limits_{\delta}\int\limits_{\tau}\int\limits_{\theta}\exp[-i(\theta t+\tau\omega+\theta\delta)]\phi(\theta,\tau)f^{*}\left(\delta-\frac{\tau}{2}\right)f\left(\delta+\frac{\tau}{2}\right)d\theta\, d\tau\, d\delta \quad ,(7)$$

where
$P\ (t,\ \omega)$ is the energy intensity;
t - is the current moment in timeж
ω is the frequency;
f is the Lamb wave signal;
f^* is the complex conjugate of f;
ϕ (θ, τ) is the characteristic function depending on $f\ (t)$.

In practice, instead of direct time-frequency analysis, some variants of equation (7) are more popular, such as the short-time Fourier transform, the Winger-Ville distribution, and the wavelet transform.

In particular, the wavelet transform uses a wavelet with a portion of the waveform that is limited in time. The average amplitude of such a portion of the wave is zero. The time-dependent signal is mapped into a two-dimensional representation with scale and time. The scale of such a representation can be related to the frequency by defining the scale value at which the scalogram reaches its maximum.

With the wavelet transform analysis, the dynamic wave signal can be examined using a localized fragment to fully display the hidden characteristics. The hidden characteristics include trends such as breakpoints or discontinuities and self-similarity. Continuous wavelet transform and discrete wavelet transform are two typical forms of wavelet transform. For Lamb wave signal, in general, continuous wavelet transform is especially effective for analysis and visualization. On the other hand, discrete wavelet transform is more useful for signal denoising, filtering, compression and feature extraction.

5.2. Results of the calculation model

The hybrid of the analytical model and its hardware implementation included a nonlinear system that consisted of four sets of equations, with each piezoelectric actuator (P_i, $i = 1, \ldots, 4$) acting in turn as an actuator based on its own reference frames, the mathematical solution of which led to the location of the damage (see Fig. 1). The wave packet transit time technique enables time-reversed imaging for wave-based fault detection. The Lamb wave governing equations in an ideal (lossless, time-independent) structure contain only second-order time derivatives.

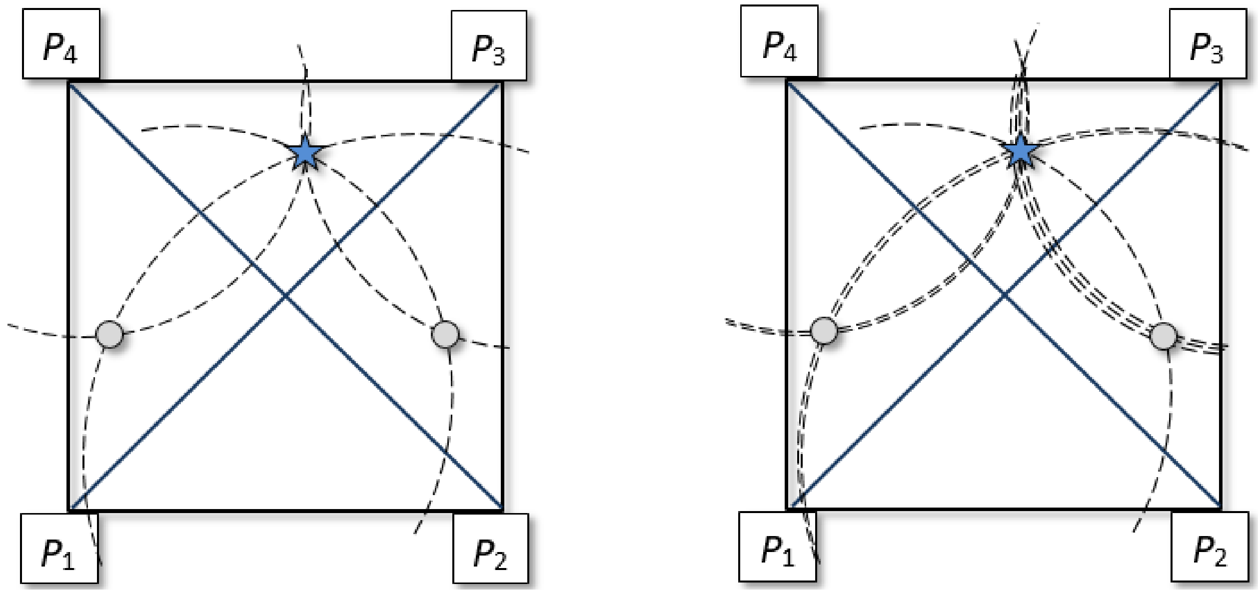

Figure 1 - Detection of delamination location: blue stars - diagnostic results; gray circles - accompanying pseudo-results; left figure - diagnostics by 4 pathways; right figure - diagnostics by 8 pathways

The set of any waves that are generated by the source and subsequently scattered, reflected, and refracted by the fault can be mapped to another set of waves. The second wave packet can exactly replicate all paths and converge synchronously at the original source, as if time were running backwards.

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Due to the fact that the composite material is generally inhomogeneous, the scattered Lamb waves measured by different actuator-sensor paths can be time-reversed, which is realized by replacing the actuator and sensor and vice versa. In this case, the Lamb wave should propagate from the sensor to the actuator. All these time-reversed wave signals, each of which exhibits a time delay due to the presence of the fault, will converge simultaneously at one point, namely the scattering point (of the fault). The results of applying the equivalent time reversal technique to localize mechanical damage in a composite plate are illustrated in Fig. 2.

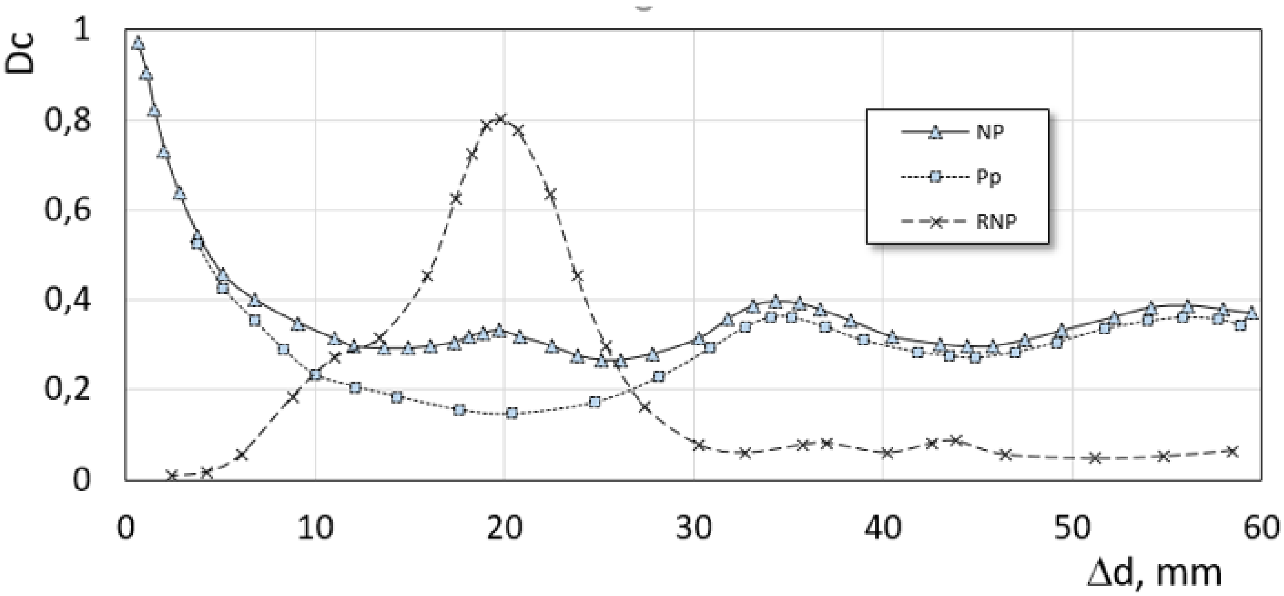

Figure 2 - Damage identification using time-reversal approach. NP – correlation notched plate; Pp – correlation perfect plate; RNP – ratio notched plate; Dc – correlation

The theoretical (phase velocity) and experimental (group velocity) values for the S0 and A0 modes in carbon fiber reinforced epoxy composite laminates for the case where two Lamb wave modes travel with different velocities in different directions (d_1) and frequencies (f_1 = 1MHz, f_2 = 0.8 MHz) are illustrated in Fig. 3 - 4.

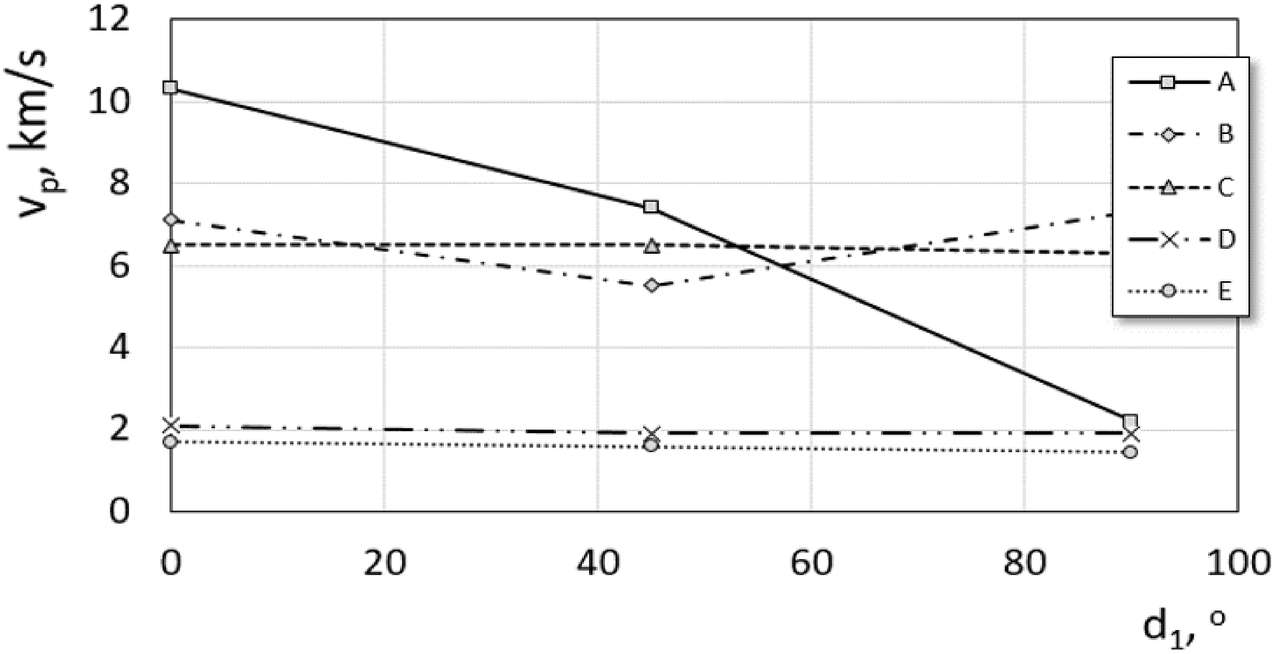

Figure 3 - Phase velocities of S0 modes in CF/FP composite laminates (for f_1): *A* – $[0]_8$, *B* – $[0/90]_{2s}$, *C* – $[\pm 45/0/90]_s$, *D* – $[0/90]_{4s}$, *E* – $[\pm 45/0/90]_{2s}$

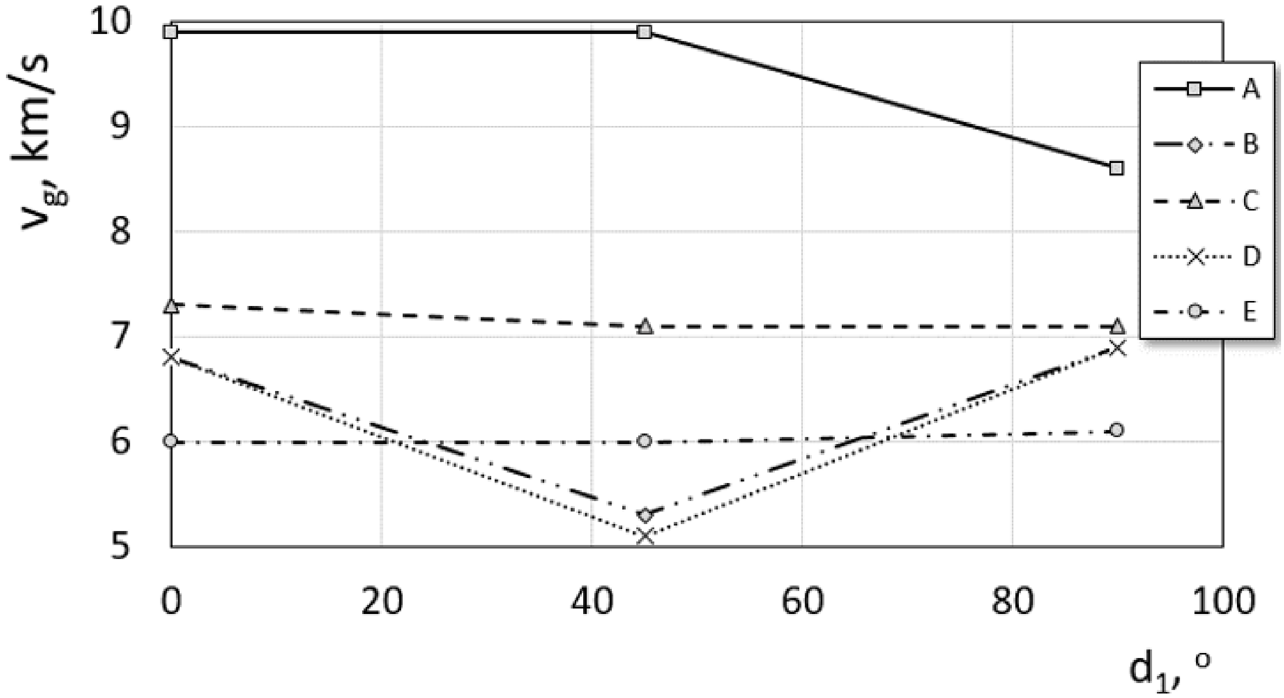

Figure 4 - Group velocities of A0 modes in CF/FP composite laminates (for f_2): *A* – $[0]_8$, *B* – $[0/90]_{2s}$, *C* – $[\pm 45/0/90]_s$, *D* – $[0/90]_{4s}$, *E* – $[\pm 45/0/90]_{2s}$

Summary and conclusions.

The difference in the directions of Lamb wave propagation is described by the retardation profile, which is a function of the reciprocal of the direction-dependent propagation velocity, 1/cg(θ) (where θ is the direction of wave propagation relative to 0°). The lowest order modes (S0, A0 and SH0) behave quite differently in different directions of propagation relative to the 0° fiber, but they all become almost directionally independent in a laminate of quasi-isotropic configuration (e.g., [±45/0/90]s). Detection of matrix cracking in samples with different crack density, as well as evaluation of the elastic modulus decay of cracked samples and comparison with those obtained during ultrasonic Lamb wave propagation testing, is usually carried out by analyzing composite samples in tension.

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