KAPITEL 10 / *CHAPTER 10* [10]
WAVELET’S STRUCTURAL DIAGNOSTIC ANALYSIS OF COMPOSITES

DOI: 10.30890/2709-2313.2025-37-01-007

Introduction

One of the most widely used composite materials in various industrial fields are laminated unidirectional subthreshold composites. Such composites are characterized by multi-layer structure and general anisotropy, which determines that the failure mechanisms of these composite structures are more complex than those of conventional structures [1, 2]. Among the main types of damage to composite structures, interlaminar delamination, which has been shown to develop from matrix microcracks in a nonlinear manner under cyclic loading, usually leads to a significant decrease in mechanical properties and ultimate failure of composite structures. The abrupt increase in the damage concentration of the volume of composite materials due to interlaminar delamination poses a serious safety hazard and undermines the reliability of composite structures. Therefore, interlaminar delamination must be detected at an early stage of development to prevent further deterioration of the structure's characteristics and to avoid serious accidents during maintenance [3].

Model-based damage detection methods are designed to reflect the damage evolution process in composite materials by establishing a theoretical model and introducing necessary variables to adjust the model to the real application scenario. Most of the model-based methods are derived from a large number of experimental studies, and many damage models have been defined by strength degradation, stiffness degradation and energy dissipation of composite materials, respectively [4, 5].

A typical example of isogeometric analysis is the method of propagating delamination propagating in straight and curved planes and buckling-delamination using the continuum shell element method. Laminated composites subjected to high-cycle fatigue loading and the corresponding delamination growth in a mixed mode are well described by a damage model including bilinear and linear-polynomial softening

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Number of characters: *19726*
Author's sheets: *0,49*

 ISBN 978-3-98924-080-3

laws. The combination of experimental and analytical studies on the unidirectional propagation of initial delamination indicates that further research is needed to extend them to flat structures, which are usually subjected to fatigue damage in practice. In this process, changing the object of analysis will lead to the fact that the adjustment of the model will become more selective, which will significantly reduce the model's ability to generalize and make it difficult to adapt to other scenarios.

Methodologies based on the analysis of the mechanical characteristics of composite specimens are becoming more and more viable in composite applications with the rapid development of Lamb wave packet-based structural monitoring technologies along with other non-destructive testing methods [6, 7]. Traditional wavelet transform-based structural monitoring methods typically focus on a specific wave packet of the received guided wave signals. This single wave packet typically corresponds to a specific S0 or A0 mode of the guided wave. Properties such as the amplitude and time of flight of this wave packet are then evaluated to identify damage. To estimate the change in the amplitude of the first wave packet passing through the composite volume and caused by damage, the so-called damage index was proposed. The defect localization and determination of its size is carried out by analyzing the transmission and reflection of S0 and A0 modes in composite laminates [8]. However, the geometry of real composite structures can be so complex that there may not always be a clear first wave packet, or the wave packet purely corresponds to the S0 or A0 mode for analysis. Therefore, a new approach is needed that does not depend on the properties of a particular wave packet.

In order to overcome the above problems, a new Lamb wave-based damage detection method for composite structures is proposed by integrating convolutional computational networks with continuous wavelet transform. In this method, continuous wavelet transform is adopted to map the time-dependent Lamb wave signal into a two-dimensional representation with scale and time. The dynamic Lamb wave signal is interrogated by a localized fragment to display its hidden characteristics such as trends, breakdown points or discontinuities, and self-similarity. In the subsequent step of this technique, a convolutional computational network is built to extract low-level features from dispersion graphs through convolutional layers and combine them into rich features for damage detection. Then, the delamination in composite structures is quickly detected by analyzing the topology of the Lamb wave monitoring network. The significant advantage of this method is that it can effectively and accurately detect the damage information of composite structures and can be easily used without excessive feature extraction and reduction.

10.1. Semi-analytical model of composite deformation

Detection of matrix cracking in samples with different crack density, as well as evaluation of the elastic modulus decay of cracked samples and comparison with those obtained during ultrasonic Lamb wave propagation testing, is usually carried out by analyzing composite samples in tension. This variational model based on mechanical stress testing is the most developed model for predicting matrix crack density. The lower limit of the elastic modulus E_c of a damaged laminate is determined in this model as follows

$$\frac{1}{E_c} \le \frac{1}{E_c^0} + \frac{1}{E_2} k_2^2 \eta(\lambda) \frac{<\chi(\delta)>}{<\delta>}, \quad (1)$$

where

$$\eta(\lambda) = \frac{3\lambda^2 + 12\lambda + 8}{60}, \quad (2)$$

$$k_2 = \frac{\sigma_2}{\sigma_0}, \qquad \lambda = \frac{t_1}{t_2}, \quad (3)$$

t_1 and t_2 are the half of the thickness of 0° and 90° layers, respectively;

σ_2 is stress in 90^0 layers;

σ_0 is the total stress in laminate.

The numerical values of k_2 (relative stiffness of 90° layers) as well as E_c^0 (initial stiffness of the laminate) are determined using the classical lamination analysis of an intact composite laminate taking into account the longitudinal and transverse elastic moduli of the composite layers. This analysis is carried out on the basis of the following system of definitions

$$\delta = \frac{a}{t_2}, \tag{4}$$

$$\chi(\delta) = 2\alpha\beta\left(\alpha^2 + \beta^2\right) \cdot \left[\frac{\cosh(2\alpha\rho) - \cos(2\beta\rho)}{\alpha \sin(2\beta\rho) + \beta \sinh(2\alpha\rho)}\right], \tag{5}$$

$$\alpha = q^{1/4} \cos\left(\frac{\theta}{2}\right), \tag{6}$$

$$\beta = q^{1/4} \sin\left(\frac{\theta}{2}\right), \tag{7}$$

$$\tan\theta = \sqrt{\frac{4q}{p^2} - 1}, \tag{8}$$

where

a is the half of the distance between two adjacent cracks;

p and q are the factors that depend exclusively on the layup and mechanical properties of the composite laminate.

A semi-analytical model is often used to obtain the stresses required to induce matrix cracking with different crack densities in specimens. Such a model for describing progressive matrix cracking in composites typically follows a specific scenario for cracking initiation and assumes a regular pattern of cracks created in 90° layers. According to this method, it is assumed that the initial cracks occur in the center of the specimen, then two subsequent cracks occur at both ends of the specimen and subsequently any new cracks occur between the two previous cracks in the bulk of the composite structure. In the case of controlled crack initiation, the crack density is related to the applied stress in the 90° layer based on the following equation

$$\left(\frac{\rho_c}{k}\right)^2 = C_{in}\left(\frac{\sigma_0}{\sigma_{in}} - 1\right), \tag{9}$$

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where

ρ_c is the crack density;

σ_{in} is the in-situ strength of 90^0 layers;

σ_0 is the stress in 90^0 layers of the multi-layered composite specimen;

C_{in} is the material-independed factor that is evaluated by fitting the theoretical master curve to the tensile test data;

K is the shear lag parameter which is given for the cross-ply composite specimen as

$$k = \sqrt{\frac{3(d+b)E_c G_{12} G_{23}}{dbE_1 E_2 (bG_{23} + d_{12})}}, \qquad (10)$$

where

b and d are the thickness of 0^0 and 90^0 layers of the laminate, respectively;

E_c is the elastic modulus of composite laminate;

E_1 and E_2 are the longitudinal and transverse elastic modulus of each composite ply, respectively;

G_{12} and G_{23} are the in-ply and out of plane shear modulus, respectively.

The propagation of Lamb waves in composite structures with anisotropic properties is accompanied by such nonlinear phenomena as the difference in group and phase velocities, as well as the direction-dependent velocity. The wave motion in an N-layer laminated composite is governed by the Navier displacement equation by applying appropriate boundary conditions in each layer.

10.2. Wave packet propagation model

The sum of the gradient of the scalar potential of the compression wave (ϕ) and the curl of the vector potential of the shear wave (ψ), for example, $u = \nabla\varphi + \nabla \times \psi$, can be interpreted as a displacement vector. The problem of wave packet propagation can, under certain restrictions, be reduced to a two-dimensional version. In this case, the correct set of such $\psi' = (0, 0, \psi)$ potentials that satisfies the boundary conditions of a

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plate medium and thus governs the propagation of a Lamb wave can be expressed as

$$\varphi_1^s = \{a_1 \exp[i\eta(x_1 - x_2)] + a_2 \exp[i\eta(x_1 + x_2)]\}, \quad (11)$$

$$\psi_1^s = \{b_1 \exp[i\eta(x_1 - x_2)] + b_2 \exp[i\eta(x_1 + x_2)]\}, \quad (12)$$

where

a_1, a_2 are the compression wave characteristics;

b_1, b_2 are the shear wave amplitudes.

For constants η and β we get

$$\eta^2 = k_l^2 - k_c^2, \quad (13)$$

$$\beta^2 = k_l^2 - k_s^2, \quad (14)$$

where

k_l is the lamb wavenumber;

k_s is the shear wavenumber;

k_c is the compression wavenumder.

The dispersion dependence for *k* has the form

$$k_s(\omega) = \frac{\omega}{\sqrt{\mu/\rho}}, \quad (15)$$

$$k_c(\omega) = \frac{\omega}{\sqrt{\frac{\lambda + 2\mu}{\rho}}}. \quad (16)$$

For the case of linear viscoelastic composites, it is more convenient to use complex values of the corresponding characteristics. The propagation of Lamb wave packets in viscoelastic materials is described by a complex version of the equation for antisymmetric modes

$$\frac{\tanh(\hat{\eta} h)}{\tanh(\hat{\beta} h)} = \frac{4\hat{k}_l^2 \hat{\eta}\hat{\beta}}{\left(2\hat{k}_l^2 - \hat{k}_s^2\right)^2}. \quad (17)$$

For shear and compression velocities we get

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$$\left(\frac{\hat{c}_s}{\hat{c}_c}\right)^2 = \frac{1-2\nu}{2-2\nu}. \quad (18)$$

In turn, the displacement on the plate surface ($x_2 = h$) in a viscoelastic composite material is equal to

$$\hat{u}_2(i\omega, x_1) = \left[\hat{\eta} A_1 \cosh(\hat{\eta} h) - i\hat{k}_l B_2 \cosh(\hat{\beta} h)\right] \cdot \exp(i\hat{k}_l x_1), \quad (19)$$

where for the complex Lamb wavenumber we get

$$\hat{k}_l(\omega) = \alpha(\omega) - i\beta(\omega), \quad (20)$$

and

$$\hat{\eta}^2 = \hat{k}_l^2 - \hat{k}_c^2. \quad (21)$$

The displacement component u_2 at two locations along propagation direction x_1 with a distance L from each other is

$$\frac{\left\|\hat{u}_2\left(x_1^2, \omega\right)\right\|}{\left\|\hat{u}_2\left(x_1^1, \omega\right)\right\|} = \exp[\beta(\omega) L]. \quad (22)$$

where || || denotes the absolute value of a complex number. On the other hand, the real part of the Lamb wavenumber, is

$$\alpha(\omega) = \frac{\omega}{c_l}. \quad (23)$$

where the Lamb wave phase velocity (c_l) is obtained from the experiments.

The complex shear modulus is defined as

$$\hat{G}(\omega) = G'(\omega) + iG''(\omega), \quad (24)$$

where

$G'(\omega)$ is the storage shear modulus;

$G''(\omega)$ is the loss shear modulus.

Complex shear modulus satisfies the expression

$$\hat{G}(\omega) = \frac{\rho\omega^2}{\hat{k}_s^2}. \quad (25)$$

 ISBN 978-3-98924-080-3

Dispersion dependencies for *G′*, *G″* and η have the form

$$G'(\omega)=\rho\omega^2\frac{1}{\left[\alpha_s^2(\omega)-\beta_s^2(\omega)\right]}, \tag{26}$$

$$G''(\omega)=2\rho\omega^2\frac{\alpha_s(\omega)\cdot\beta_s(\omega)}{\left[\alpha_s^2(\omega)-\beta_s^2(\omega)\right]^2}, \tag{27}$$

$$\eta=\tan\delta=\frac{2\alpha_s(\omega)\cdot\beta_s(\omega)}{\alpha_s^2(\omega)-\beta_s^2(\omega)}. \tag{28}$$

The formation of matrix cracks in polymer matrix composite laminates depends on various factors such as layup, cross-sectional strength of composites, impact strength and rigidity of the polymer matrix, strength, rigidity of fibers, etc. The experimental study of the mechanical damage field in the volume of the composite material must necessarily be accompanied by an assessment of the crack density in the composite samples. Such an assessment is necessary due to the fact that the base of some cracks was not fully opened, and some cracks did not grow in a straight line. To avoid any uncertainties, cracks crossing the entire area of 90o layers were counted at a fixed distance approximately in the middle of the sample volume.

Figure 1 illustrates the average induced crack densities ρ_c (mm^{-1}) in the test set of specimens for different applied stress levels θ_L (MPa), predicted by the elastic model and also obtained from tensile tests.

Lamb wave multipath scattering is effectively used for mechanical damage monitoring in laminar composites. In Lamb wave-based structural health monitoring, it is common to pre-record baseline signals when the structure is free of damage. On this basis, the residual field U^{rs}, which subtracts these baseline signals from the measured signals, isolates the effects of the unknown damage introduced between the two measurements. In the single scattering approximation, the residual field only takes into account the direct scattering path of the damage. Therefore, the following relation holds

$$U^{rs}(\omega;s,r)=F(\omega)G_0(\omega;\|u-s\|)\psi(\theta;\theta_1,\theta_2)G_0(\omega;\|r-u\|), \tag{29}$$

 ISBN 978-3-98924-080-3

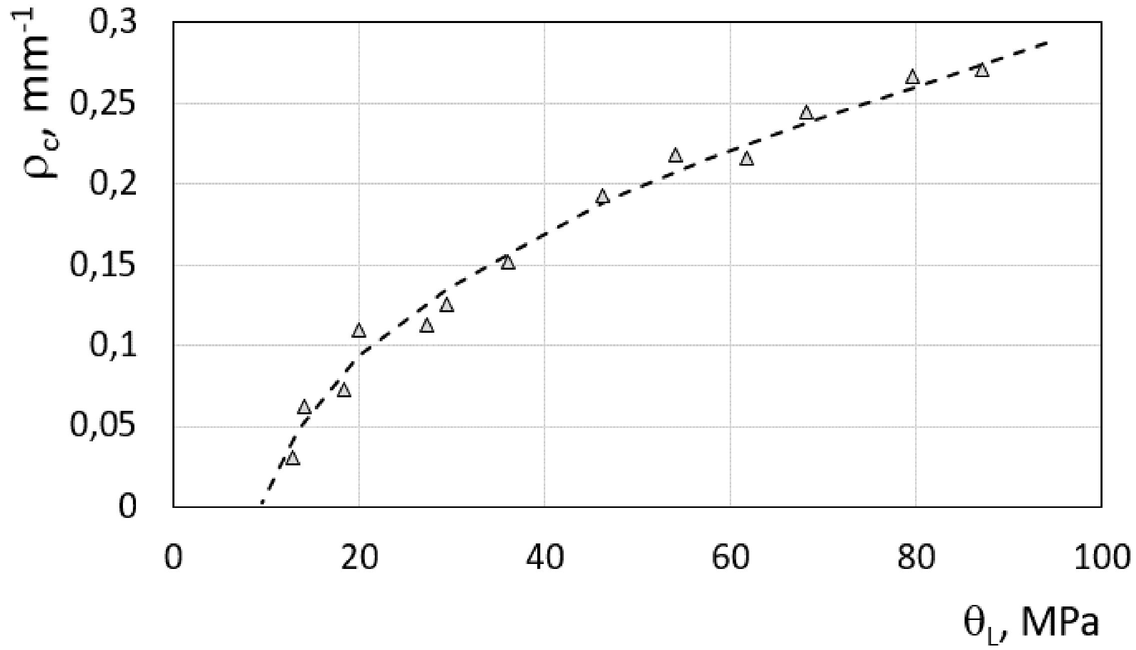

Figure 1 - Dependence of crack density on stress level in 90⁰ layers

$$G_0(\omega;d)=\frac{\exp[ik(\omega)d]}{\sqrt{\frac{d}{d_r}}},\qquad (30)$$

where

$F(\omega)$ is the frequency domain excitation;

ψ (ω; θ_1, θ_2) is the part of scaterring pattern;

$s = [s_x,s_y]^T$ is the location of transmitter;

$r = [r_x,r_y]^T$ is the location of receiver;

$u = [u_x,u_y]^T$ is the location of damage;

θ_1 is the incoming angle;

θ_2 is the outcoming angle;

d is the actual propagation distance;

d_r is the reference distance;

$k = \omega/c_p$ is the wavenumber;

c_p is the phase velocity.

Obstacles that can scatter Lamb waves are associated not only with sources of unknown damage, but also with previously known features in the structure, such as

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edges, stiffeners, lap joints, and rivets. The residual signal may also include waves that are scattered multiple times between these known scatterers and the target (i.e., the damage).

The entire set of known singularities in composite structures can be classified as linear reflectors (e.g. ribs and stiffeners) and point scatterers (e.g. rivets) according to their dimensions. An additional improvement of the above technique is related to the inclusion of multipath scattering characteristics between linear reflectors and damage in the analysis area.

Fermat's principle can be applied to the propagation of acoustic rays. This principle leads to the presence of six possible propagation paths, since the transmitter $s = [s_x, s_y]^T$ and the receiver $r = [r_x, r_y]^T$ are present near two adjacent edges of the structure (i.e. the damage is located at $u = [u_x, u_y]^T$).

Analysis of the dispersion curves of the Lamb wave packet modes for reflectors uniformly distributed over the side surfaces of a laminated composite sample indicates that each linear reflector can be considered as a mirror, creating a virtual transmitter or receiver in a symmetrical position. The location of this virtual transmitter or receiver is determined by the position of the actual and mirror, and is independent of the location of the damage.

Summary and conclusions.

The semi-analytical model of wavelet transforms proposed in this work allows one to detect deformations of a fairly wide range of sizes and shapes in the volume of a laminar composite. The basic assumption of the model is to consider defects in laminated composite structures as linear reflectors and point scatterers of appropriate sizes. It was found that the propagation of Lamb waves in composite structures with anisotropic properties is accompanied by such nonlinear phenomena, in particular, different group and phase velocities, the numerical values of which are determined using the wavelet transform. Monitoring of mechanical damage in laminated composites was performed using Lamb wave multipath scattering. The analysis showed that in the single scattering approximation the residual field takes into account only the direct damage scattering path.