KAPITEL 3 / *CHAPTER 3* [3]
MAIN IMPACT PARAMETERS FOR MULTI-ENERGY STATIC MODELING

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Introduction

Decarbonization has become a global energy priority, with the EU and other regions setting ambitious climate neutrality targets. This shift is accelerating the deployment of Renewable Energy Sources (RES), such as wind and solar, and phasing out fossil-fuel power plants. Recent geopolitical tensions have further emphasized the need for energy security and reduced dependency on imports.

However, the variability of RES and limited flexibility in electricity systems—due to constrained storage and modest demand-side management—pose operational challenges. Grid reinforcements to manage peak RES output are often costly and underutilized. Meanwhile, electrifying all sectors, especially hard-to-abate industries, remains technically and economically difficult. Alternative energy carriers like hydrogen and heat networks are gaining attention for their flexibility and storage potential.

Integrating these carriers through a Multi-Energy (ME) approach enables more efficient system operation, planning, and decarbonization. Static ME models—focused on steady-state analysis—support this integration by capturing synergies across electricity, gas, hydrogen, and heat systems. These models help optimize dispatch and infrastructure investment while addressing growing public resistance to large-scale grid expansions. This paper provides a tutorial introduction to static ME modeling, outlining core modeling principles, integration methods, numerical challenges, and practical insights for researchers and system planners. It highlights the growing policy momentum for multi-vector planning at the EU level and the essential role of ME modeling in shaping resilient, low-carbon energy systems.

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3.1. Energy carriers static modeling approaches

Before delving into the most widely used approaches for Multi-Energy (ME) system modeling, it is essential to first analyze the mathematical representations of the primary components associated with each energy carrier. This foundational analysis starts from comprehensive, general modeling formulations and then identifies simplifications necessary for enabling the simulation of large-scale systems—typically involving thousands of nodes per energy carrier.

Two fundamental characteristics must be preserved to ensure computational feasibility and robustness of these models: **model convexity** and **linearity**. Convexity ensures that an optimization problem yields a unique global solution, thereby eliminating the risk of convergence to suboptimal local minima. This property is particularly important in time-sequenced simulations, where inconsistent or incomparable solutions across time steps could otherwise undermine the reliability of system-wide assessments. Linear models are inherently convex, and their structure allows for the use of powerful and efficient solution techniques such as **interior-point algorithms**, which are particularly well-suited for solving large-scale problems. In the context of energy system planning, many models are framed as **Mixed Integer Linear Problems (MILPs)**. This is largely due to the presence of binary decision variables used to represent discrete investment choices—such as whether to build new infrastructure components. MILP formulations offer a significant computational advantage over **Mixed Integer Nonlinear Problems (MINLPs)**, which tend to be considerably more complex and computationally intensive. For example, the **FlexPlan** project, a major European research initiative, relies on MILP modeling to optimize long-term transmission system planning under uncertainty and flexibility constraints.

More broadly, in the modeling of Multi-Energy Systems (MES), the overall complexity of the mathematical model is directly tied to the level of detail used in representing individual system components. Increasing model fidelity—by incorporating technical constraints, nonlinear dynamics, uncertainty, or auxiliary services—inevitably leads to more complex equations and, in many cases, the need for

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specialized solvers. As such, a critical trade-off arises between model accuracy and computational tractability. Researchers and system planners must carefully balance these aspects to ensure that the model is both representative of the real system and solvable within available computational resources.

An insightful classification of modeling complexity in ME systems is provided in [1], where MES models are divided into **seven categories**, each characterized by increasing levels of mathematical intricacy. The associated outlines key modeling challenges for each category, clearly illustrating how model complexity escalates with the incorporation of features such as nonlinearities, technical operating limits, stochastic inputs, demand flexibility, and the inclusion of both short-term operational and long-term investment variables.

The sections that follow examine the mathematical structure of static models for the most relevant energy carriers—electricity, compressible fluids (including natural gas and hydrogen), and thermal energy networks. Among these, **compressible fluid networks** and **heat networks** present the greatest modeling challenges. The governing equations for such systems are inherently nonlinear and not easily reducible to linear forms without introducing significant approximations. Furthermore, unlike electricity networks, where static modeling techniques are well established and widely adopted, modeling approaches for gas, hydrogen, and thermal networks are still evolving and lack standardization. For these reasons, the discussion on compressible fluid networks is given particular emphasis. This extended analysis explores the general formulation of the equations governing gas and hydrogen flow and systematically evaluates the simplifications needed to obtain linear models suitable for inclusion in integrated ME simulations. While most multi-carrier modeling efforts involve some degree of approximation to enable tractable computation, it remains essential to understand the theoretical underpinnings of these simplifications and their implications on model fidelity and interpretability.

By establishing a solid foundation in the mathematical modeling of each energy vector, this analysis sets the stage for a deeper exploration of how these components can be effectively integrated into holistic, scalable Multi-Energy models for system

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simulation, optimization, and planning.

3.2. Electricity networks modeling

A systematic literature review was undertaken to establish a robust foundation for further analysis, encompassing both scientific and gray literature on the conceptual framework of sustainable companies. Scientific sources were drawn from multidisciplinary databases, specifically Web of Science (WoS) and Scopus, employing targeted search criteria centered around corporate sustainability (CS) and associated concepts, as detailed in [2] of the Supporting Information (SI). The search parameters were restricted to peer-reviewed, English-language journal articles published between 2013 and 2023, ensuring accessibility to full-text versions.

Electricity networks are among the most extensively modeled systems in energy simulation studies. Numerous well-established modeling approaches have been developed over the years, tailored to a variety of simulation domains such as system planning, real-time operation, and contingency analysis. As a result, the foundational methodologies for representing electric grids in different contexts are widely recognized and adopted within the research and industrial communities. A broad range of simulation tools and open-access libraries exist to support such modeling efforts. Notably, **MATPOWER** for the MATLAB programming environment [3] and **PowerModels** for the Julia language [4] are among the most widely used platforms, offering accessible and modular frameworks to build power system optimization and simulation applications. Despite this maturity, a concise overview of electricity network modeling remains essential within this discussion—for the sake of completeness relative to other energy carriers and to highlight the specific assumptions and simplifications applied in **static modeling** contexts.

For a more thorough understanding of electric power system modeling, readers are encouraged to consult specialized textbooks and reference works [5]. However, the following section outlines some of the core principles and common modeling choices relevant to static electricity network models. One of the fundamental elements of power system modeling involves the computation of **active and reactive power flows** across network branches. A simplified representation of power lines is typically adopted by neglecting the shunt elements in the π-equivalent circuit model , which reduces computational complexity while retaining sufficient accuracy for many planning and operational applications.

In **distribution networks**, the assumption commonly applied to transmission networks—that the line reactance is much greater than resistance—no longer holds true. As such, the **DC power flow approximation**, which neglects reactive power and line losses, is not generally applicable. Instead, the **DISTFLOW** formulation offers a more appropriate alternative for distribution systems. This method incorporates reactive power calculations through suitable approximations, enabling the system to remain within a linear solvable structure [6].

An alternative linearization method, applicable primarily to transmission networks, involves the use of **Power Transfer Distribution Factors (PTDFs)**. These coefficients represent the incremental change in active power flow on a specific line lll resulting from an additional power injection at a given node nnn. PTDFs enable a linear approximation of how power redistributes across the network in response to varying injections, and are especially useful for modeling **meshed transmission systems**. In such networks, the PTDF values typically range between 0 and 1, depending on the relative impedance of the paths—where a value of 0 indicates no impact from a node's injection on a given line, and 1 indicates a full transfer through that line. In contrast, **distribution networks**, which are typically radial in structure, exhibit PTDF values of either 0 or 1, reflecting their tree-like topology. The complete PTDF matrix has dimensions equal to the number of lines by the number of nodes and can theoretically be derived from the inverse of the system's impedance matrix [7]. However, in practice, PTDF values are usually determined through **sensitivity analyses** conducted on specific network scenarios.

Once established, the PTDF matrix can be incorporated into system constraints to enforce **transmission line capacity limits**. While this representation is particularly effective in **fixed-topology studies**—such as those concerned with system dispatch and electricity market outcomes—it is less suitable for **grid expansion planning**. This is because the addition or removal of transmission lines alters the network’s impedance ratios, and consequently, the PTDF values themselves, which would need to be recalculated for every new scenario. The static models used in electricity system studies can be further enriched by incorporating additional operational constraints. These might include limits on **maximum charge or discharge capacity**, **power ramp rates**, and **degradation effects** due to repeated cycling of equipment such as batteries [8]. The inclusion of such features enhances model fidelity but also increases its complexity. Therefore, especially in large-scale **planning models**, a balance must be struck between capturing essential system behaviors and maintaining **numerical tractability**. To this end, the general recommendation is to **minimize the number of constraints** while preserving the most critical dynamics relevant to the study’s objectives. When integer variables—typically associated with investment decisions—are included, it becomes even more imperative to ensure that the model remains **linear** to allow efficient solution using standard MILP solvers. This approach enables the simulation of expansive systems with acceptable computation times, even when dealing with high-dimensional decision spaces.

3.3. Compressible fluids modelling

Natural gas networks are intricately interconnected with electrical systems, primarily due to the reliance of gas-fired thermoelectric power plants on natural gas for electricity generation. Moreover, the integration of synthetic gas into natural gas infrastructures further emphasizes the operational interdependence between these energy vectors. One of the central equations employed in modeling gas flows within pipelines under steady-state conditions is the Weymouth equation, which describes pressure drops along gas pipelines. This relationship, typically applied by neglecting gravitational effects (a common simplification in large-scale gas system simulations), forms the backbone of many static gas flow models. While the Weymouth equation is inherently non-linear, it is possible to apply linearization techniques to approximate its

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behavior and facilitate its integration into broader optimization models, as explored in [9]. However, the Weymouth equation does not account for dynamic phenomena, particularly the delayed propagation of pressure disturbances along pipelines, which becomes critical in long-distance gas transmission systems—such as trans-European networks. In such cases, a non-stationary approach is required to accurately capture the time delay between gas injection at the source and its availability at the consumption point.

To this end, the method of characteristics is commonly employed to solve the dynamic equations that govern gas flow in non-steady conditions. Although this method introduces time dependency into the model—seemingly at odds with a static modeling framework—it is indispensable for capturing wave propagation effects and modeling pressure transients resulting from sudden changes in injection or extraction rates. This quasi-dynamic treatment is particularly relevant for gas systems with extensive spatial coverage, where the decoupling between injection and delivery is non-negligible. By contrast, such effects are far less significant in district heating networks due to their limited geographic scale.

Conceptually, the method of characteristics discretizes the pipeline into spatial segments of length Δx and computes time steps Δt based on the time required for forward and backward pressure waves (along the characteristic curves C^+ and C^-) to traverse that segment. This approach, governed by the Friedrichs–Courant condition, offers a highly accurate solution for calculating mass flow rates and pressure drops, assuming the isothermal gas flow hypothesis holds—an assumption generally valid except in fast-transient conditions.

Nonetheless, this methodology presents several practical limitations. First, the governing equations are highly non-linear and impose strict requirements on the time discretization. This dependency necessitates solving the entire spatial-temporal equation set simultaneously, which poses significant computational challenges for large-scale system studies. As a result, the method of characteristics is rarely used in large static models, where simplified representations such as the Weymouth equation—or its linearized versions—are typically preferred.

Alternative linear formulations for modeling gas dynamics have been proposed. One particularly notable approach [9] involves applying finite difference methods to a predefined spatial–temporal grid, combined with Taylor series-based linear approximations. While this yields a linearized system, it does not guarantee preservation of wave propagation speeds, primarily because the spatial-temporal discretization is not aligned with the characteristic curve slopes. Furthermore, as with the method of characteristics, values at future time-space points are calculated iteratively based on prior and current values. This again implies that, for optimization purposes, the full equation set must be simultaneously formulated and solved, making it ill-suited for static models at scale.

Beyond the modeling of pipeline flow, compressor stations and pressure regulation valves are additional key components in gas transmission systems that exhibit non-linear behavior. These elements must be carefully modeled, particularly in optimization problems such as network planning, where system-wide operational feasibility is evaluated over long time horizons.

Compressor stations, positioned along transmission lines to maintain adequate pressure levels, typically consist of multiple compressor units, scrubbers, cooling systems, emergency shutdown mechanisms, and sophisticated automated control systems. Most compressors are powered by gas turbines that consume a portion of the transported gas—estimated to be around 3–5% of the flow—and represent approximately 20% of total operational costs [10].

3.4. Heat networks modeling

Modern distribution networks are undergoing a profound transformation. Historically designed as radial structures for unidirectional power flow—from centralized generation sources to medium and low-voltage consumers—they are now evolving into active networks due to the growing integration of distributed energy resources (DERs). One of the most significant contributors to this shift is the proliferation of photovoltaic (PV) installations, which introduces both opportunities and challenges, particularly concerning the variability and intermittency of solar generation.

To address these challenges, there is a growing emphasis on the deployment of local flexibility resources. These include flexible loads, small-scale energy storage systems, and the thermal inertia of large heating infrastructures connected to the distribution grid. Notable examples include numerous residential and commercial swimming pools in Denmark [3], as well as extensive district heating networks in urban areas. Such systems possess an inherent capacity to absorb fluctuations in electricity supply, thereby enhancing the stability and resilience of the grid.

Among these, district heating systems (DHSs) are particularly promising due to their inherent thermal storage capabilities. Since water used in DHSs can be distributed across a range of permissible temperatures, these systems can be operated flexibly, behaving as controllable thermal loads. This flexibility makes DHSs valuable assets for sector coupling, i.e., the coordinated operation of electricity and thermal energy systems. Accordingly, it becomes critical to develop integrated modeling frameworks capable of analyzing and exploiting the synergies between electrical distribution grids and district heating networks.

According to the system representation outlined in [5], the steady-state behavior of a district heating pipeline can be characterized using the following equation: The Weymouth Equation, which models pressure drops along the pipeline. This equation, commonly applied to compressible fluid flow, is similarly valid here, especially since mass flow rate remains constant under steady-state conditions. A heat propagation equation, which models the transfer of thermal energy along the pipeline by analyzing an infinitesimal volume element. Although thermal transients within DHSs can be neglected in many static simulations—given their relatively small geographic scale and the use of large time steps (typically hourly or more)—properly capturing the flexibility contribution of such systems necessitates accounting for thermal energy storage within the pipelines. This is typically achieved by replacing the static heat propagation model with an integral energy balance, yielding a lumped-parameter representation of thermal dynamics.

This lumped model allows for an adequate representation of the system's thermal inertia, which is crucial when assessing its ability to shift loads and respond to electrical system needs in real-time. In addition to pipelines, another essential component in district heating systems is the heat load, which typically refers to heat exchangers where thermal energy is transferred to end-users. These are generally modeled using a simple heat transfer relationship that quantifies the exchanged thermal power based on the temperature difference between the supply and return lines. For a more in-depth discussion of the various modeling approaches, tools, and challenges associated with heat network simulation, the reader is referred to comprehensive reviews available in [11]. These works provide valuable insights into both steady-state and dynamic modeling techniques, as well as an overview of software platforms and simulation environments developed for this purpose.

Conclusion

Static modeling of Multi-Energy Systems (MES) plays a vital role in supporting energy system planning, enabling the integration of electricity, gas, hydrogen, and heat networks. While electricity systems benefit from mature modeling approaches, compressible fluids and heat networks pose greater challenges due to their nonlinear nature. Linearized static models—particularly MILP formulations—offer a practical trade-off between accuracy and scalability. The analysis confirms that maintaining model convexity and tractability is crucial for simulating large-scale systems and informing investment decisions. Despite simplifications, such models effectively capture intersectoral synergies and flexibility potential, supporting the transition toward resilient and low-carbon energy infrastructures. Future research should focus on standardizing models for gas and thermal networks and incorporating uncertainty for more robust planning under dynamic conditions.