KAPITEL 4 / *CHAPTER 4* [4]
DISPERSION OF SYMMETRIC AND ANTISYMMETRIC MODES IN MECHANICALLY STRESSED COMPOSITE PLATES

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Introduction

Numerous studies have been conducted in the field of non-destructive evaluation to improve reliability and reduce life cycle costs for current and future infrastructure. Theoretical studies are based on techniques using natural frequencies and mode shapes, dynamic time-domain responses, or guided waves. Among the various non-destructive testing methods, Lamb waves have been shown to be sensitive tools for detecting most types of defects, effective for detecting small and subsurface damage, and capable of inspecting large areas [1]. Understanding the scattering characteristics of Lamb waves plays an important role in the successful application of Lamb waves for damage detection. Various studies have been conducted to study the interaction of Lamb waves with different types of damage in different materials [2, 3].

It should be noted that the use of fiber-reinforced composite laminates has been steadily increasing in various engineering applications, due to their high specific stiffness, light weight, and corrosion resistance compared to traditional metallic materials. However, failure of composite laminates is more critical than failure of metallic materials because there are more failure modes (e.g., delamination, matrix cracking, and fiber rupture) and these failures are more difficult to detect and characterize. Delamination is one of the most common and serious failure modes in composite laminates [4]. It can be caused by low-velocity impacts, manufacturing process defects, or fatigue loading. Delamination is the separation of adjacent subsurface laminations without any obvious visual evidence on the surface. The critical extent of delamination depends on several factors, in particular, the design philosophy and the type of material. Factors such as the anisotropic nature and multilayer

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characteristics of composite laminates make analytical solutions for Lamb wave scattering at delamination difficult. However, several studies have been conducted to study and understand Lamb wave scattering at delamination using numerical simulations.

In particular, a 2D finite element method with plane strain assumption was used to study the reflection of the fundamental symmetric (S0) Lamb wave from delamination in unidirectional and cross-laminated laminates [5, 6]. It was shown that the S0 Lamb wave cannot be used to detect delamination at through-thickness locations with zero shear stress. An analysis of the reflection characteristics of S0 and A0 Lamb waves as a function of delamination in cross-laminated laminates using the 2D strip element method showed that the A0 Lamb wave is sensitive to delamination at all through-thickness locations. The 2D finite element methodology can be used to explain the interaction of the A0 Lamb wave with delamination [7]. It is shown that the S0 Lamb wave is generated at the leading edge of delamination due to mode conversion. However, it is limited to only the sub-laminates in the delamination region and converts back to the A0 Lamb wave when leaving the delamination.

These studies have improved the understanding of the fundamental physics of Lamb wave interactions during delamination. They have also shown that the Lamb A0 wave is sensitive to delamination at all locations through the thickness. In addition, since the Lamb A0 wave has a shorter wavelength than the Lamb S0 wave at the same frequency, it is potentially more sensitive to delamination. Due to the favorable characteristics of the Lamb A0 wave, it has attracted interest in the context of damage detection. The same can be said for the symmetric as well as the antisymmetric modes of this wave. The aim of this study is to provide an extended analysis of the dynamics of the propagation of symmetric and antisymmetric Lamb wave modes in the volume of a laminated composite in the context of non-destructive testing for the presence of mechanical damage in the volume of the composite structure.

4.1. Wave curves of Lamb wave modes in laminated composites

The group velocity of the SH0 and S0 modes has pronounced dispersion characteristics. However, even greater dispersion is observed for symmetric modes in the quasi-isotropic laminate [+45/45/0/90]s. In contrast, the dispersion of the A0 mode in both laminates is weaker beyond $\omega h/c_T = 1$ (where ω is the frequency, h is the composite sample size and c_T is the characteristic wave velocity). This feature is effectively used for structural monitoring of laminar composites.

Characteristic wave curves, including velocity, as well as slow Lamb wave curves, propagate in composites at a given frequency. Most of these curves are centrosymmetric about the origin. This feature is a consequence of the fact that the fiber orientation of each individual plate is invariant when h is replaced by $h + p$ (p = const). Moreover, all characteristic wave curves change with frequency due to the dispersion nature. Characteristic wave curves can be constructed at $\omega h/c_T = 4$ with two symmetric modes (S0 and SH0) and three antisymmetric modes (A0, SH1 and A1). The wavelengths of these modes at a fixed $h = 30$ are much larger than the fiber diameters and the distance between the fibers. The set of characteristic wave curves considered can vary significantly for different frequencies due to its dispersive nature.

Such a phenomenon as energy focusing is observed for volume waves in anisotropic solids. It should also be noted that energy focusing is also observed for SH wave modes. The deceleration curves have many inflection points. This implies that the same direction of the group velocity can correspond to several directions of wave propagation. The specific directions are determined by the features of the wave front of the Lamb wave packet.

The characteristic frequency of wave packet propagation in thin quasi-isotropic laminated composite is lower than the cutoff frequencies of A1 and S1 modes. Therefore, only fundamental modes (A0, S0 and SH0) exist in thin laminate. The angular dependence of Lamb waves in the laminate [+45/45/0/90] becomes weaker due to quasi-isotropic stacking.

Numerical analysis revealed that the A0 mode has a maximum along 45^0

directions, since the bending of the dominant outer plate is oriented in these directions. The wave curve for each layer of the composite material has no inflection. This fact is a consequence of the quasi-isotropic stacking, not the dispersion characteristics.

Since the velocity curves are approximately independent of the wave propagation direction, the average wavelengths can be estimated using solutions of the characteristic equations. These wavelengths are found to be comparable to the plate thickness, but much larger than the fiber diameters and the fiber spacing. This also shows that the wavelength of A0 is shorter than that of the S0 and SH0 modes.

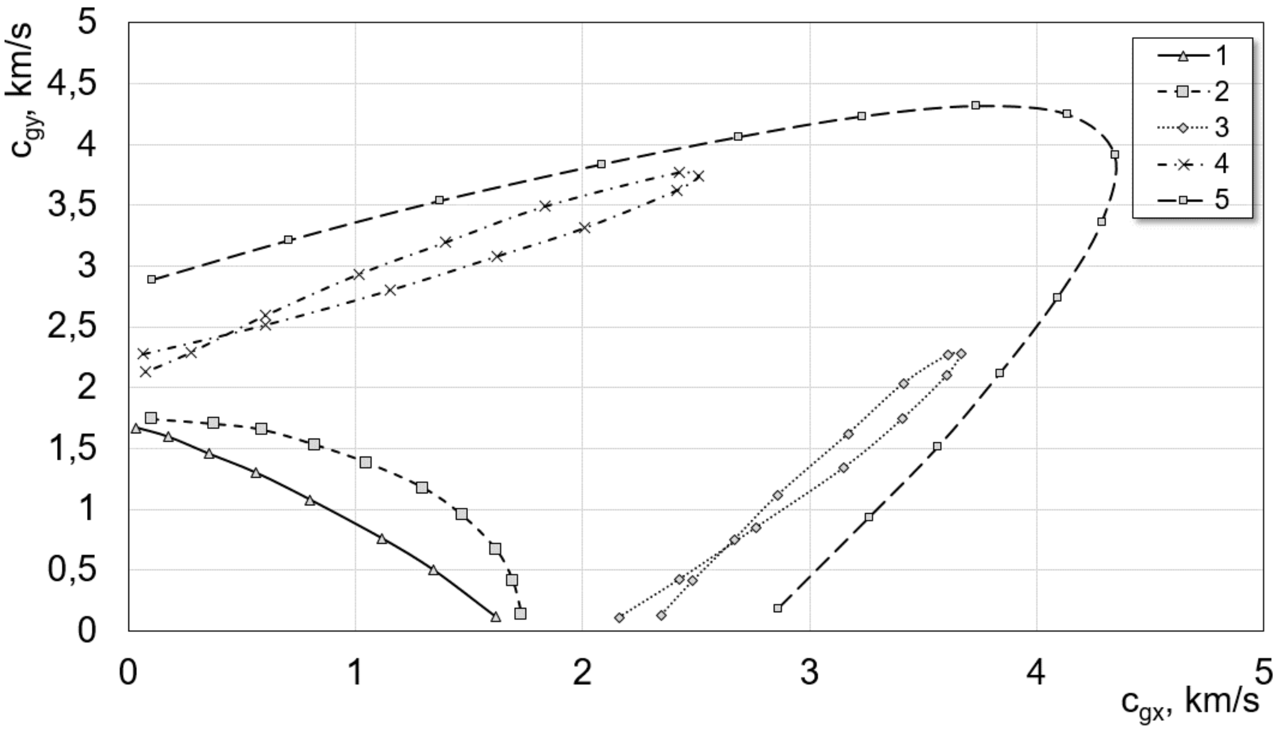

Figure 1 - Wave curves in the laminate [+456/-456]: *1 – SH1; 2 – A0 (theory); 3 – SH1A; 4 – SH1B; 5 – A1*

Comparison results between theoretical prediction and experimental measurement of Lamb waves in a relatively thick [+45$_6$/ 45$_6$]s laminate are shown in Fig. 1. It can be seen from the figure that the exact solutions agree satisfactorily with the experimental results for both symmetric and antisymmetric modes. Even higher modes such as S1 and A1 can be detected experimentally.

The SH0 and S0 modes are difficult to distinguish in the very low frequency range

(50 – 150 kHz). This feature is explained by the fact that the difference in arrival time between the two modes is very small. It can be concluded that the two modes have already appeared even before the end of the excitation duration. It should be noted, however, that the exact solution for the S2 modes is not consistent with the experimental results. This difference is determined by the presence of larger scattering signals from inhomogeneities in the high-frequency range for lower wave modes than from the S2 mode itself.

A comparison of the Lamb wave results in the quasi-isotropic [+45/45/0/90]s laminate shows that for excitation frequencies up to 1 MHz, the eight-layer thinner composite exhibits only fundamental guided waves (SH0, S0 and A0) that propagate in the laminate. It should also be noted that the exact solutions are in good agreement with the experimental measurements for both the group velocity dispersions and the wave curves. Detection of mechanical damage in composites requires the combined use of both the Lamb wave method and the basic concepts of the theory of elasticity and elasticity in non-isotropic media, which in general include laminar composite structures. The consideration of waves with a fixed wave front shape implies flat deformation conditions in the *x-z* plane, i.e. $\varepsilon_y = \varepsilon_{xy} = \varepsilon_{yz} = 0$, and negligible changes in non-vanishing strains and stresses in the y direction. The basic relations of elasticity theory are the linear stress-strain equations for a rotated composite layer and are given by

$$\sigma = [C]\varepsilon, \tag{1}$$

where

$\boldsymbol{\sigma} = \{\sigma_x, \sigma_z, \sigma_{xz}\}^T$ is the extended stress vector;

$\boldsymbol{\varepsilon} = \{\varepsilon_x, \varepsilon_z, \varepsilon_{xz}\}^T$ is the extended strain vector.

The ply stiffness matrix [**C**] is

$$[C] = \begin{bmatrix} C_{11} & C_{13} & 0 \\ C_{31} & C_{33} & 0 \\ 0 & 0 & C_{55} \end{bmatrix}. \tag{2}$$

For each point in the volume of a sample made of a composite structure, the equilibrium equations for stresses in the x and z directions can be written. The equilibrium equations, in turn, are reduced to equations describing the motion of a wave with a fixed wave front shape. The derivatives of mechanical stresses with respect to y are negligibly small. As a result, the equilibrium equations for mechanical stresses take the form

$$\sigma_{x,x}(x,z,t)+\sigma_{xz,z}(x,z,t)=\rho\,\ddot{u}(x,z,t), \tag{3}$$

$$\sigma_{xz,x}(x,z,t)+\sigma_{z,z}(x,z,t)=\rho\,\ddot{w}(x,z,t), \tag{4}$$

where

u is the displacement component along *x* axis;

w is the displacement component along z axis;

ρ is the average density of the lamina composite.

The thickness displacement variations are satisfactorily described by the kinematic hypothesis of the layer-by-layer theory. The layer-by-layer theory allows for the presence of piecewise (zigzag) fields across the thickness of the composite. The calculation method is simplified by assuming that a typical laminate can be divided into *N* discrete layers. Each discrete layer can contain one layer, one sublayer, or two sublayers. Consequently, the displacement field in the laminate takes the form

$$u(x,z,t)=\sum_{n=1}^{N}U^{n}(x,t)\Psi^{n}(z), \tag{5}$$

where

U^n are the in-plane displacement of fixed layer along *x* axis;

$\Psi^n(z)$ are the linear interpolation functions.

The Fourier transform of the generalized displacement vectors *U* (x, t) and *W* (*x*, *t*) for a wave propagating along the *x*-axis of the strip is performed first with respect to the time variable *t*, and then with respect to the spatial variable *x*, following the standard rule

$$\left\{\tilde{U}(x,\omega),\tilde{W}(x,\omega)\right\} = \int_{-\infty}^{\infty} \{U(x,t),W(x,t)\}\exp(-i\omega t)dt , \tag{6}$$

$$\left\{\tilde{U}(\xi,\omega),\tilde{W}(\xi,\omega)\right\} = \int_{-\infty}^{\infty} \{U(x,\omega),W(x,\omega)\}\exp(-i\xi x)dx , \tag{7}$$

where

ω is the circular frequency;

ξ is the axial wavenumber.

The 4*N*-dimensional first-order problem for a given real value of frequency ω and wave number ξ can be formulated as follows

$$[A(\omega) - \xi B(\omega)]\hat{V} = \hat{P} . \tag{8}$$

Non-trivial solutions of the homogeneous part of equation (8) are obtained by imposing a boundary condition, the essence of which is that the determinant of the coefficient matrix of the vector *V* must be equal to zero.

This is equivalent to the characteristic dispersion relation of the equation. For a given frequency ω, the characteristic equation has *m* = 1, ... , 4*N* complex eigenvalues $\xi^m = \xi^m_{Re} + i \cdot \xi^m_{Im}$. The complex solutions are the axial wave numbers for all modes existing at the excited frequency.

Among all the derived modes, there are propagating, inhomogeneous, and decaying modes. Accordingly, these modes are characterized by real, complex, and purely imaginary axial wave numbers.

4.2. Lamb wave modal decomposition

The modal decomposition allows us to decompose the eigenvector *V* in terms of the right eigenvector φ_m as follows

$$\hat{V} = \sum_{m=1}^{4N} V_m \varphi_m = \hat{P} . \tag{9}$$

The generalized coefficients V_m can be expressed using the following relationship

$$V_n = \frac{\psi_n^T \hat{P}}{(\xi_n - \xi) D_{mn}}, \tag{10}$$

where

$$D_{mn} = \psi_m^T B \varphi_m. \tag{11}$$

Accordingly, the resulting Green's function in the domain of frequencies and wave numbers has the form

$$\hat{V} = \sum_{m=1}^{4N} \frac{\psi_m^T \hat{P}}{(\xi_m - \xi) D_{mn}} \varphi_m. \tag{12}$$

The frequency band fluctuations are superimposed on changes in the vector of the external force F, which occurs during mechanical displacement. With such superposition, various loading cases can arise. The most common in the deformation of laminar composites are two types of surface loads: a concentrated force at the point $x = x_0$ and a distributed shear force, which is applied over a finite length. The dependence of the force on time can be any. The force vector can be expressed using the following relationship

$$f(x,t) = F_0 \delta(x - x_0) f(t), \tag{13}$$

where

F_0 is the amplitude of the force F;

$\delta\ (x - x_0)$ is the Dirac delta-function;

$F\ (t)$ is the time dependence of external force.

The double Fourier transform in both time and space variables yields a transformed force vector

$$\hat{F}(\xi, \omega) = F_0 \frac{1}{2\pi} \exp(i\xi\, x_0) F'(\omega), \tag{14}$$

where F’(ω) is the transformed forcing vector.

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Applied composite mechanical stress is

$$\tau_{xy}(x,t)=\tau_0\left[H(x+\alpha)-H(x-\alpha)f(t)\right] \quad (15)$$

The shift in the frequency-spatial domain obtained using the inverse Fourier transform has the form

$$\widetilde{V}(x,\omega)=\frac{1}{2\pi}\sum_{m=1}^{4\pi}\int_{-\infty}^{\infty}\frac{\psi_m^T\hat{P}}{(\xi_m-\xi)D_{mn}}\varphi_m\exp(i\xi x)d\xi \quad (16)$$

The integral in equation (16) is evaluated numerically using the Cauchy residue theorem. The integrand has singularities only where the denominator is zero, i.e., for the poles $\xi = \xi_m$. Only those poles that lead to propagating Lamb waves are analyzed. Therefore, it is necessary to calculate the residues from these N_d poles

$$\widetilde{V}(\xi,\omega)=-i\sum_{m=1}^{N_d}\frac{\psi_m^T\hat{P}}{D_{mn}}\varphi_m\exp(i\xi_m x) \quad (17)$$

For a given excitation frequency ω, equation (17) represents the frequency response of the system. To decide which poles produce the correct propagating waves, we calculate the group velocity for each eigenvalue from equation

$$c_{gm}=\frac{\psi_m K_\xi \varphi_m}{2\omega_m\psi_m M\, x_m} \quad (18)$$

Verification numerical calculations using the features of Lamb wave propagation in the composite volume were performed according to the following model. A pair of concentrated normal forces of equal magnitude was applied to the upper and lower surfaces of the sample in two different configurations.

The first configuration corresponded to the unidirectionality of the applied forces. In this case, the generation of antisymmetric wave modes was assumed. The second configuration included oppositely directed external deformation forces. Forces of this direction excited symmetric wave modes. In addition, the calculations were performed under the assumption that the time dependence of the applied load f (t) was modeled by a Gaussian 3-cycle sinusoidal tone burst.

The predicted time response of the fringe at a distance comparable to one third of the specimen length, measured from the point of application of the external force, was calculated in terms of the u and w displacement components. The calculated results showed good agreement with the experimental results for the propagation of the A0 and S0 waves. It should be noted that the widely used semi-analytical finite element method leads to worse predictions for the identification of mechanical damage in the volume of the composite.

Transit time of each mode is consistent with the predicted group velocity. The S0 mode wave has a high dispersion at the selected excitation frequency. It should be noted that the comparison of the predicted wave response will be sensitive to small errors in the numerical calculation of the wave characteristics. In addition, a small oscillatory component is observed at the beginning of the time response. The initial times are compared with the arrival time of the guided wave mode, which is introduced by the inverse Fourier transform applied to obtain the results.

The use of generalized models of layered laminates allowed us to obtain semi-analytical solutions for the directed propagation of a direct Lamb wave with a fixed profile in laminated composite strips. The model describing the propagation of Lamb wave packets in composite inhomogeneous structures is solved analytically in axial propagation. Three-dimensional theory of layered laminates was used to analyze the variation of displacement with thickness. A double Fourier transform transformed the problem into the frequency-wavenumber domain, where the modal properties of the structure were extracted. Finally, two inverse Fourier transforms provide a solution to the problem in terms of displacements.

The proposed finite difference stress excitation models work effectively and provide very good excitation of Lamb waves at fixed frequencies for both isotropic plates and quasi-isotropic composite laminates. A single piezoelectric sensor can excite both S0 and A0 modes simultaneously. Alternatively, with multiple sensors evenly spaced over the side surface of a laminated composite sample, either S0 or A0 can be excited separately. Numerical calculations with the finite difference model, which uses the 3-D method with displacement and shear force excitation, indicate that the group velocities of the A0 mode corresponding to one and several sensors are significantly different. However, there is no such difference for the S0 mode.

Summary and conclusions.

Analysis of the dispersion curves of Lamb waves indicates the need for preferential use of antisymmetric modes A0, SH1 and A1 in the process of continuous monitoring of both nucleation processes and developed deformation kinetics for areas of the laminated composite volume located at a significant distance from the lateral surface. Therefore, the calculation of the propagation velocities of antisymmetric modes in the presence of oppositely directed external forces applied to the laminar composite is also associated with an easier computational process. The modal properties of the composite structure can be determined with high accuracy using the double Fourier transform, which shifts the focus of the analysis to the frequency-wavenumber domain.